# Two decades review of reliability-based topology optimization developments

## Revue de deux décennies de développements d'optimisation fiabiliste de topologie

Ghais Kharmanda[1,2,*], Hasan Mulki[3]

[1] Mechanics Laboratory of Normandy, INSA Rouen, France
[2] 3D printing 4U (UG), Cologne, North Rhine-Westphalia, Germany
[3] College of Engineering and Technology, American University of the Middle East, Kuwait
* Corresponding author: Ghais Kharmanda, 3D printing 4U (UG), Nördlinger St 10, 51103 Cologne, Germany, Ghais.Kharmanda@3d-printing-3u.com

**ABSTRACT.** This review seeks to classify the different developments of the Reliability-Based Topology Optimization (RBTO) according to their formulations and methods. According to the RBTO formulations, two standpoints (topology and reliability standpoints) are generally treated, while according to methods, they are divided into loop and mixed methods. The different trends and gaps are discussed later. The selected publications over the last two decades (from December 2001 to December 2021) contain the term RBTO in their title and restricted to English language. There are several other publications, which have been written in other languages and some others containing the integration of the reliability analysis into topology optimization, but the RBTO was not included in their titles. These two types of publications are unfortunately not included in this review since it is the objective to provide the readers (especially the new researchers) a good guide (map) to the different RBTO developments and applications. Some additional publications considering other objective and constraint functions and dealing the RBTO with microstructural levels are also included since they were realized under the same period and containing the term of RBTO in their title. The first publication of the Reliability-Based Topology Optimization model was a technical report at Aalborg University in Denmark, published in December 2001. It was a new model where many critical discussions appeared between the topology optimization community and the reliability one during 2002 and 2003 (in several meetings and conferences). That led to delay the appearance of the first journal articles until 2004. According to the author's knowledge, more than 70 journal papers and more than 25 other publications (conference papers, chapters, reports... etc.) have been found during the last two decades. Almost 50% of journal papers have been published during the last four years which means that there is a strong interest to implement this model.
**KEYWORDS.** Reliability Analysis, Topology Optimization, Reliability-Based Topology Optimization, Uncertainty, Optimization Methods.

### 1. Summary of work

In the most real engineering design applications, uncertainty can be considered on load (direction and magnitude), material properties (elastic modulus), and geometric parameters and also on manufacturing and operating processes. The measurement classification is based here on four criteria: applicability, efficiency, robustness, and capacity. Regarding the first measurement classification (according to developed RBTO formulations), it can be noted that the most developments of Reliability-Based Topology Optimization (RBTO) in literature can be divided into two groups. The first group called developments from a topology optimization standpoint, which leads to various layouts with decreasing rigidity (increasing compliance) levels which is considered as a disadvantage of these methods. Moreover, some researchers consider that there is no physical meaning when representing the limit state function by the prescribed volume constraint. Nevertheless, the second group being called developments from a reliability analysis standpoint, often provides us with the same layouts with increasing rigidity (or decreasing compliance) levels. The single disadvantage of these methods is to lead to the same layouts with different thickness. Some researchers consider that this result does not represent any value since a detailed design stage is required to control the structural rigidity. However, when integrating additive manufacturing, the drawback of this standpoint can be reduced since there is no need to perform shape and/or sizing optimization procedures.

On the other hand, when considering the other measurement classifications (according to the developed RBTO methods), the efficiency and the robustness are the main criteria to consider when selecting a method to use. So, the first method category concerns the looped methods when the whole RBTO process is treated with loops. Here, we have single loop methods, decouple loop methods and nested loop methods. In addition, to increase the efficiency and/or robustness, some techniques such as simulation (ex. Monte Carlo simulation, approximation (Response Surface Method) ... are integrated to increase the robustness levels.

Figure 1 shows a distribution diagram of 72 journal papers (blue color) and 28 other publications (orange color) have been published during the last two decades. So, there is a strong increase during the last four years.

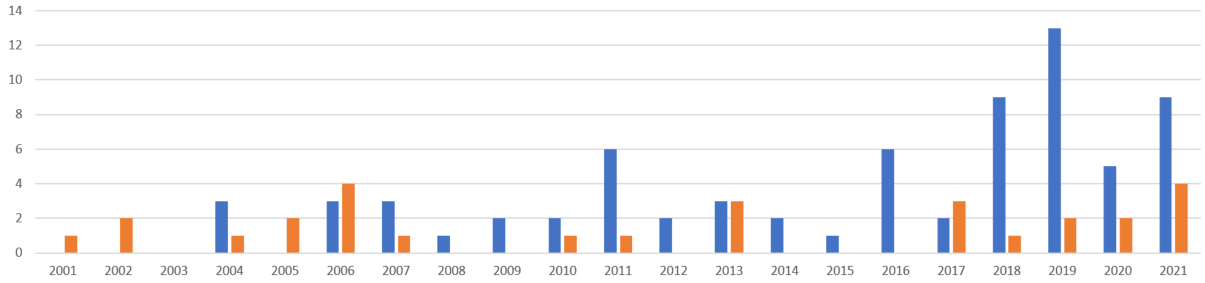


**Figure 1.** *Distribution diagram of RBTO publications over two decades*

Some of these papers contain new methods and techniques, while the other only contain applications. So, in the next section, we present a classification according to the used RBTO formulations, while the classification according to the developed methods or techniques will be discussed later where several flowcharts can be found in order to establish a detailed comparison of the advantages and disadvantages regarding four criteria such as robustness, generality, efficiency and capability.

## 2. Comparative study based on the reviewed work

### 2.1. *Classification according to RBTO formulations*

The topology optimization is an important topic in the optimum design field where it is the objective to provide the best material distribution in the studied structures. Several concepts can be integrated into the topology optimization with the object of improving its role during the design process. One of these concepts is the structural reliability. This integration leads to a new model, so-called Reliability-Based Topology Optimization (RBTO) model where several solutions with special advantages can be produced (Kharmanda and Olhoff 2001; Kharmanda and Olhoff 2002; Kharmanda et al. 2004; Kharmanda et al. 2007a and b).

The majority of RBTO developments can be grouped in two main standpoints:

**From a topology optimization standpoint**, Kharmanda and Olhoff (2001) have developed an RBTO model with object of supplying the designer with several reliability-based structures, while the classical topology optimization (Deterministic Topology Optimization: DTO) gives only a single deterministic topology. Their simple model utilized the reliability index approach of the probabilistic model in order to handle aleatory uncertainties. During first five/six years of developments (Kharmanda and Olhoff 2001; Kharmanda and Olhoff 2002; Kharmanda et al. 2004;

Kharmanda et al. 2007a; Kharmanda et al. 2007b), continuum topology optimization was performed under uncertainties of material properties and external loads using the First-Order Reliability Method (FORM). It has been demonstrated that the importance of the RBTO model generates more reliable structures than those generated by DTO for same weights. In this RBTO strategy, reliability constraints have been added into initial DTO problems in a simple way to obtain different layouts (Kharmanda and Olhoff 2001). The initial step is the sensitivity analysis which is utilized to show the impact of random variables on the compliance. The goal in this step is to pick the random variables that have a big impact on the objective function. A special Gradient-Based Method (GBM) was developed here where the limit state function is supposed to be a linear combination of the random variables. Using this approach, two separate sequential steps are considered. The first step is represented by a sensitivity study to pick the most effective random parameters, whereas the second step is to execute the optimization procedure which itself utilizes a sensitivity study with respect the optimization parameters represented by the material densities. Next, Kogiso et al. (2006) applied the RBTO to frame structures considering multiple modes. After that, Zhang et al. (2008) developed a level set method for the RBTO model and applied it to compliant mechanisms. Thereafter, Kogiso et al. (2009) utilized Single-Loop-Single-Vector (SLSV) method for the RBTO model and applied it to frame structures considering multiple performance criteria with the object of improving the convergence properties. In the work of Patel (2010), several RBTO applications were presented in order to design meso-structures. In the same period, Silva et al. (2010) performed the RBTO model in a single-loop approach using KKT condition and used both component and system failure probabilities. And Kogiso et al. (2010) used SLSV method for the RBTO model and applied it to frame structures considering multiple criteria. After that, Olyaie et al. (2011) used the cell-based smoothed finite element method for the RBTO model and applied it to a linear piezoelectric micromotor. In the work of Patel and Choi (2012), a classification approach for the RBTO model was developed using probabilistic neural networks. Next, Olyaie et al. (2013) used an optimum finite element method for the RBTO model considering a linear piezoelectric micromotor. In the work of El Hami and Radi (2013), the integration of the reliability analysis to topology optimization was compared to the classical shape and sizing optimization procedures. After that, Kanakasabai and Dhingra (2014) proposed a methodology which was built upon the bidirectional evolutionary structural optimization (BESO) method for solving the deterministic optimization problem. Their proposed method is suitable for linear elastic problems with independent and normally distributed loads, subjected to deflection and reliability constraints. Next, Liu et al. (2016a) decoupled the two-layer nesting involved in RBTO using a particular optimization procedure. Thereafter, Zhao et al. (2016) used three methods (one nested loop approach and two decoupled loop approach) to solve the RBTO examples. In the same period, Liu et al. (2016b) developed a new RBTO method based on sensitivity approximation and applied it to ground structures. In the work of Papadimitriou and Mourelatos (2018), a mean-value second-order saddle point approximation was elaborated to perform the RBTO model. In a conference paper of Li et al. (2018), there are similar examples and results which can be found in the works of Kharmanda and Olhoff (2001) and Kharmanda et al. (2004). Next, Chun et al. (2019a) developed a RBTO model by ground structure method employing a discrete filtering technique. In the same period, Chun et al. (2019b) developed TOPO-Joint using the RBTO model for 3D-printed building joints. In the works of Kharmanda et al. (2019a), the effect of reliability index changes is studied on the resulting layouts. A validation was simply carried out using shape optimization procedures. And in Kharmanda et al. (2019c), the RBTO developments considering two points of view and a fatigue damage example is used to show the advantages of the topology optimization standpoint. Next, Meng et al. (2020) introduced a hybrid RBTO method to deal with epistemic and aleatory uncertainties. It was a cost-effective single optimization loop method established on KKT optimality condition. Recently, Pérez-Rúa et al. (2021) performed a simultaneous multi-objective framework for the RBTO model applied to an offshore wind arm collection system. In addition, Ma and Wang (2021) elaborated an RBTO framework for two-dimensional phononic crystal band-gap structures based on interval series expansion and mapping conversion method. Furthermore, Madruga et al. (2021) utilized evolutionary methods to apply the RBTO model to three-dimensional structures analysis.

**From a reliability analysis standpoint**, the classical topology optimization procedure is formulated as getting the stiffest structural layout considering a volume restriction. It was believed in this case that the feasibility of volume restriction is not fundamental in structural design challenges. It is then more significant to consider the stiffness variations under uncertainties. Bae and Wang (2002) were the first who initiated the developments from a reliability analysis standpoint by expressing the topology optimization as structural volume minimization with a displacement limitation and applied the Reliability-Based Design Optimization (RBDO) procedure to keep the stiffness robustness in the topology design. After that, Jung and Cho (2004) extended the work of Bae and Wang to geometrically nonlinear structures under loading and material uncertainties. In the same period, Kang et al. (2004) applied the RBTO model from a reliability analysis standpoint to electromagnetic systems and Moon et al. (2004) applied it to thermal systems considering convection heat transfer. Next, Patel et al. (2005a) and (2005b) utilized the gradient free Hybrid Cellular Automata (HCA) method to perform the RBTO model. Their formulation incorporates the uncertainty in material properties. Thereafter, Mozumder and Renaud (2006a and b) established an Investigation of different RBTO developments presenting some critical points of the topology standpoint. In the same period, Kim et al. 2006a applied the RBTO model considering reverse engineering (laser scanned model) which was applied to design of the upper part of a cellular phone. And in Kim et al. 2006b, the RIA (Reliability Index Approach) and PMA (Performance Measure Approach) were adopted to the RBTO model for static and eigenvalue problems. In works of Wang et al. (2006) and Kim et al. (2007a), the RBTO model was applied to microelectromechanical systems. Later, Kim et al. (2007b) carried out the RBTO problems using evolutionary structural optimization strategy. After that, Kang and Luo (2009) utilized convex models for non-probabilistic RBTO geometrically nonlinear structures (NRBTO). Concerning the NRBTO issues, Kang and Luo (2009) developed a new strategy considering uncertainties of loading conditions and material properties in a multi-ellipsoid convex model framework. To address epistemic uncertainties, they introduced an ellipsoid convex model to assess the uncertain-but-bounded parameters. Thereafter, Mashayekhi et al. (2011) used a two-stage optimization method for the RBTO model and applied it to double layer grids. In the same period, Yoo et al. (2011) used successive standard response surface method for the RBTO model. In addition, Nguyen et al. (2011) developed a single-loop system for the RBTO model considering statistical dependence between limit-states. They utilized matrix-based system reliability analysis with multiple finite element mesh resolutions. The matrix-based system reliability method can consider statistical dependence and evaluate parametric sensitivity in an efficient way. Furthermore, Li et al. (2011) combined evolutionary structural optimization (ESO) and the reliability analysis strategy based on the stochastic finite element method (perturbation). The stochastic perturbation method is utilized to solve the reliability to transform the implicitly bound constraints into explicit. Moreover, Cho et al. (2011) used a bidirectional evolutionary structural optimization to the RBTO model to the inner reinforcement of a vehicle’s hood. And Eom et al. (2011) employed bi-directional evolutionary structural optimization technique and the standard response surface approach to achieve the RBTO model. Thereafter, Xu (2013) established a dynamic non-probabilistic RBTO model and applied it to truss with uncertain-but-bounded parameters. In the same period, Jalalpour et al. (2013) applied the RBTO model to trusses with stochastic stiffness. In addition, Torii and Novotny (2013) used topological derivatives, sequential optimization, and reliability assessment to perform the RBTO model. And Zhao et al. (2013) applied the RBTO model to a control arm of suspension for lightweight design. Next, Luo et al. (2014) considered local failure constraints when applying the RBTO model to continuum structures. Later, Zhao et al. (2015) used stochastic response surface method with sparse grid design to perform the RBTO model. After that, Kanakasabai and Dhingra (2016) developed an efficient RBTO approach considering the bidirectional evolutionary structural optimization (BESO) method used for solving deterministic optimization problems. In the same period, López et al. (2016)

compared the deterministic versus reliability-based topology optimization and performed their different applications to aeronautical structures. Here, they combined the Sequential Optimization and Reliability Assessment (SORA) with external optimization software in order to perform Reliability-Based Topology Optimization (RBTO). Thereafter, Lee and Lee (2017) employed the RBTO model for obtaining an acoustically optimal partition layout inside a muffler. In the same period, Clark (2017) presented an analytic sensitivity approach to perform the RBTO model without considering the nested problem cases. And Bobby et al. (2017) applied the RBTO model to uncertain building systems under stochastic excitation where the structural performance is measured to control the uncertainty influence in every iteration. A simulation based RBTO method (based on Monte Carlo simulation) for uncertain structures subject to stochastic excitation based on the first excursion probabilities was considered. Furthermore, Wang et al. (2017) embedded the non-probabilistic reliability index explicitly throughout the topology optimization procedure. Interval sets were employed to limit the range of structural uncertainty, and the distance between the target plane and the failure plane was utilized as a measure of non-probabilistic reliability. Next, Kang and Liu (2018) studied the RBTO model against geometric imperfections with random threshold model. In the same period, da Silva and Beck (2018) studied the RBTO model of continuum structures under local stress constraints. It is widely recognized that these uncertainty effects should be integrated into the optimization problems to obtain robust and reliable solutions. And Yin et al. (2018) considered the fuzzy method for the epistemic uncertain behavior based on membership level. In the work of Wang et al. (2018a), a multiscale RBTO methodology was elaborated for truss-like microstructures with unknown-but-bounded uncertainties. In addition, Wang et al. (2018b) elaborated a non-probabilistic RBTO method and applied it to continuum structures with convex uncertainties. Furthermore, Zheng et al. (2018) utilized the non-probabilistic RBTO considering a multidimensional parallelepiped convex model. They developed an efficient decoupling scheme to change the double-loop NRBTO into a sequential optimization process, using the sequential optimization & reliability assessment (SORA) method associated with the performance measurement approach (PMA). And Dos Santos and Torii (2018) decoupled the nested reliability optimization problem under stress constraints into a deterministic optimization step and a reliability analysis step by sequential optimization and reliability assessment. Unfortunately, the time point where the maximum dynamic response occurs, may vary considerably during the optimization process. And López et al. (2018) also performed the RBTO model through the RBDO algorithm Sequential Optimization and Reliability Assessment (SORA) based on its robustness, accuracy, and decoupled nature. This allows to use an external optimization software when dealing with the topology optimization problem. The validation of their approach was carried out through aircraft structures. After that, Wang et al. (2019a) developed a new RBTO framework for the concurrent design of solid and truss-like material structures with unknown but bounded uncertainties. In the same period, Wang et al. (2019b) developed an RBTO approach for continuum structures under interval uncertainties. And Wang et al. (2019c) developed a non-probabilistic RBTO approach for multi-material layout design via interval and convex mixed uncertainties. In addition, Wang et al. (2019c) proposed a reliability index for seeking the optimal layout of multi-material structures with mixed uncertainties of interval and convexity. It is generally recognized that the uncertainty influence should be integrated into the optimization problems in order to obtain robust and reliable solutions (see Wang et al. 2019c; da Silva and Beck 2018). Furthermore, Wang et al. (2019d) elaborated non-probabilistic RBTO model of continuum structures considering local stiffness and strength failure. Moreover, Wang et al. (2019e) dealt with a truss layout design under non-probabilistic RBTO framework with interval uncertainties. Additionally, Wang et al. (2019f) elaborated a non-probabilistic RBTO method of compliant mechanisms with interval uncertainties. Furthermore, Vishwanathan and Vio (2019) developed an efficient quantification of material uncertainties in the RBTO model using random matrices. And Sato et al. (2019) studied the RBTO model under shape uncertainty modeled in Eulerian description. Recently, Gao and Liu (2021) applied the RBTO considering stochastic heterogeneous microstructure properties. They developed a novel framework of reliability-based topology optimization called RBTO-AIS to deal with microscale uncertainties. In addition, Wang et al. (2021a) developed a non-probabilistic RBTO scheme for continuum structures based on the parameterized level-set method and interval mathematics. Furthermore, Wang et al. 2021b presented a novel dynamic RBTO (DRBTO) approach with NTR constraints of dynamic response is elucidated for time-variant mechanical systems considering interval uncertainties. And Wang et al. (2021b) developed a novel dynamic RBTO framework for continuum structures via interval-process collocation and the first-passage theories. In addition, Wang et al. (2021c) studied the RBTO for heterogeneous composite structures under interval and convex mixed uncertainties. Also, Yin et al. (2021) proposed an efficient RBTO approach for the structural lightweight design of planar continuum structures. Moreover, Xia et al. (2021) applied the RBTO to freely vibrating continuum structures considering unknown-but-bounded uncertainties. And finally, Pham and Hoyle (2021) proposed an algorithm to find robust reliability-based topology optimized designs under a random-field material model in order to avoid the nested loop methods.

Few works have been carried out to combine between the two standpoints. In Kharmanda et al. (2019a), inverse optimum safety factor methods were developed to combine between both points of view. In the context of these approaches, Kharmanda and Antypas (2020a and b) studied the influence of geometry uncertainty on resulting RBTO layouts and showed its importance when integrating into additive manufacturing. Kharmanda et al. (2020a) integrated the RBTO into biomechanics considering the hollow stems used in cementless total hip arthroplasty. Kharmanda et al. (2020b) developed inverse optimum safety factor approaches and next applied it to bridge structures (Kharmanda et al. 2020c). In the same period, Sleesongsom and Bureerat (2020) presented a multi-objective RBTO using a fuzzy set model where an equivalent possibilistic safety index was defined along with mass, and compliance. Next, Kharmanda et al. (2021a) applied the linear and nonlinear RBTO to bridge structures and Kharmanda et al. (2021b) performed a nonlinearity RBTO investigation and applied it to total hip replacement. In addition, De et al. (2021) applied the RBTO using stochastic gradients. They minimized a weighted sum of compliance and mass subjected to a reliability constraint.

## 2.2. *Classification according to RBTO methods*

The different developed RBTO methods can be classified into two categories: loop and mixed methods. The loop methods are totally based on optimization loops, while the mixed methods are composite of iterative optimization loops and other techniques (simulation, approximation ...). Even the mixed methods can be composed of double loops (one optimization and the other for simulation, approximation ...). In this section, we model the different types of methods by generalized flowcharts.

### 2.2.1. *Loop methods:*

#### 2.2.1.1. Single loop methods

Single-loop approaches modify the reliability analysis problem so that it is coupled with the design optimization problem in a single loop. The optimum point and the MPP then converge simultaneously. Figure 2 shows a generalized flowchart of the single loop method where all parameters (deterministic and random vectors) are updated at each iteration. In certain methods, the sensitivity analysis is required when updating the optimization variables.

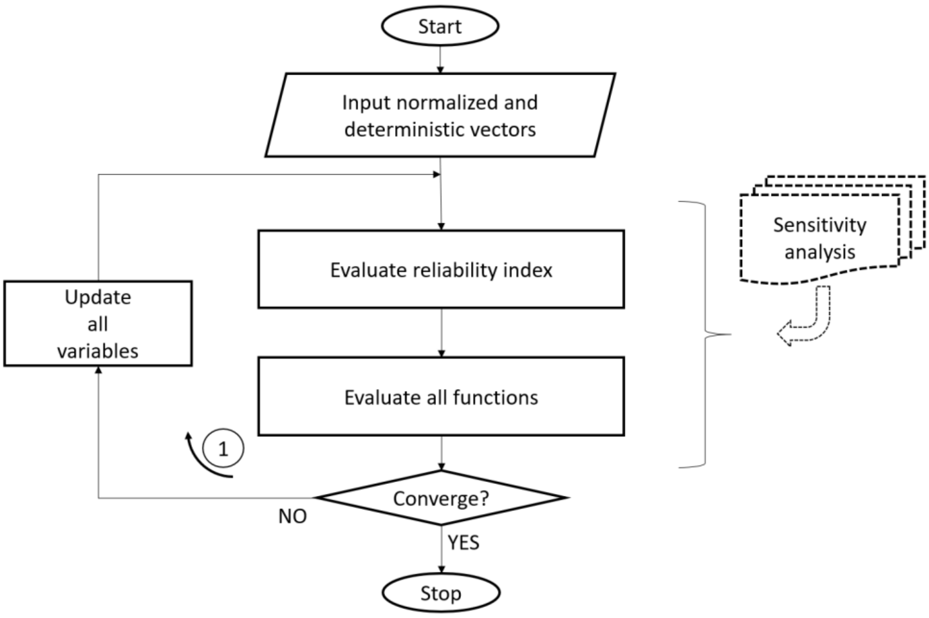


**Figure 2.** *Generalized flowchart for single loop methods*

For example, Single Loop Single Vector (SLSV) algorithm is utilized by Kogiso et al. (2009) and (2010). They presented their method in a simple iterative (line search) process, however, Wang et al. (2017) utilized non-probabilistic reliability index explicitly throughout the topology optimization process. In their optimization loop, the process is separated into several levels such as reliability assessment level, sensitivity analysis level and the used topology optimization method level.

#### 2.2.1.2. Decoupled loop methods

Decoupling approaches extend this idea further by completely dividing the reliability analysis and the deterministic design. These approaches have two successive loops: The first one is to carry out the reliability analysis and the second one is needed to perform the topology optimization problem. This method was first utilized by Kharmanda and Olhoff (2001) to implement the idea of the RBTO in a simple and rapid way. At this period, according to the authors' knowledge, the common methods for RBDO (Reliability-Based Design Optimization) were the single and nested loop methods, while this kind of methods was not known to many multi-objective optimization communities. Figure 3 shows a generalized flowchart of the decoupled loop methods where two successive loops can be found: Loop 1 is used to perform the reliability analysis, while Loop 2 is used to perform the topology optimization problem. The module of sensitivity analysis may be utilized in one or both loops according to the used optimization method.

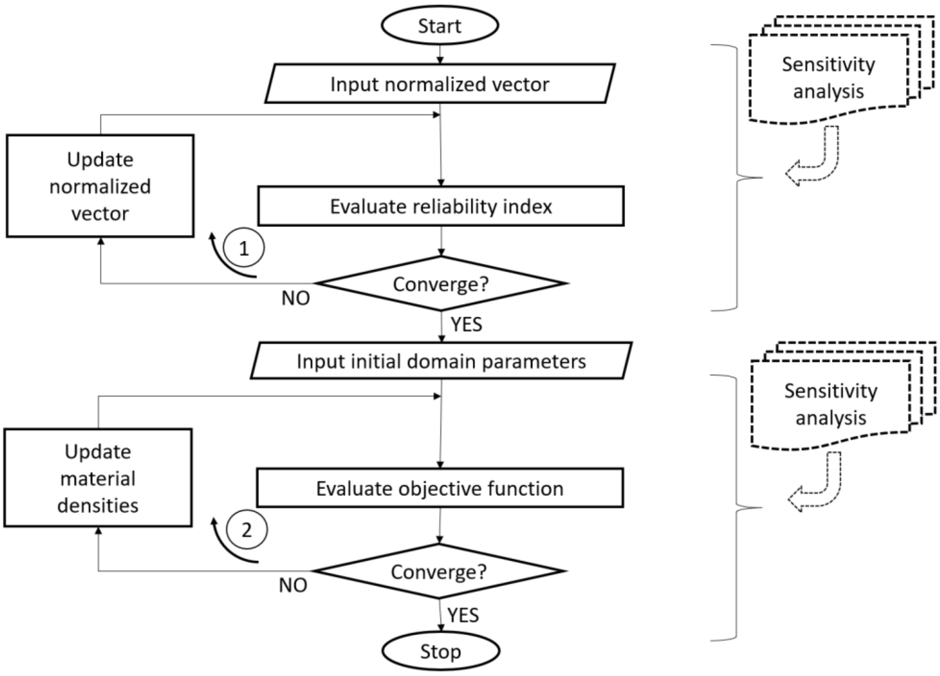


**Figure 3.** *Generalized flowchart for decouple loop methods*

Kharmanda and Olhoff (2001) proposed the GBM to compute first the normalized vector satisfying a required reliability index and next a topology optimization is performed considering the resulting normalized vector. The developed GBM-based RBTO contains two successive loops. The first one is to calculate the optimum value of the normalized vector. The resulting random vector is considered as the input data of the second loop represented a topology optimization method called SIMP (Solid Isotropic Material with Penalization). In a very simple way, the designer can generate several layouts.  Next, Zhang et al. (2008) utilized level-set method to perform the RBTO in successive loops. The reliability analysis is first performed using FORM and next the topology optimization is performed using level set method. Thereafter, Mashayekhi et al. (2011b) considered a two-stage optimization method (SIMP-ACO method) to carry out the RBTO procedure. In the same period, Olyaie et al. (2011) utilized a cell-based smoothed finite element method for RBTO. Next, the Sequential Optimization and Reliability Assessment (SORA) method is combined with external optimization (bidirectional evolutionary structural optimization) (see Kanakasabai and Dhingra (2016)).

### 2.2.1.3. Nested loop methods

This strategy is usually utilized when considering several objectives (multi-objective optimization). When considering finite element simulation, it leads to a high computation time consumption. The processes are performed in two (or more) separate spaces. At each iteration, a new space is established and then several sub-iterations are required. The problem becomes more complicated when dealing with several multi-objective optimization cases. In addition, there can be some problems of convergence stability. In certain cases, the algorithm may not converge. This kind of methods was the first to utilize when dealing with the RBDO problem over two or three decades. For real applications, these drawbacks (computing time consumption and week convergence stability) made the designers to hesitate to integrate the RBDO model in their design studies. Figure 4 shows a generalized flowchart for the nested loop methods where at each iteration, a call to another loop is required to evaluate the reliability level of the current iteration. When the sensitivity analysis module is utilized in each loop, the computing time consumption makes a real obstacle to perform the optimization process.

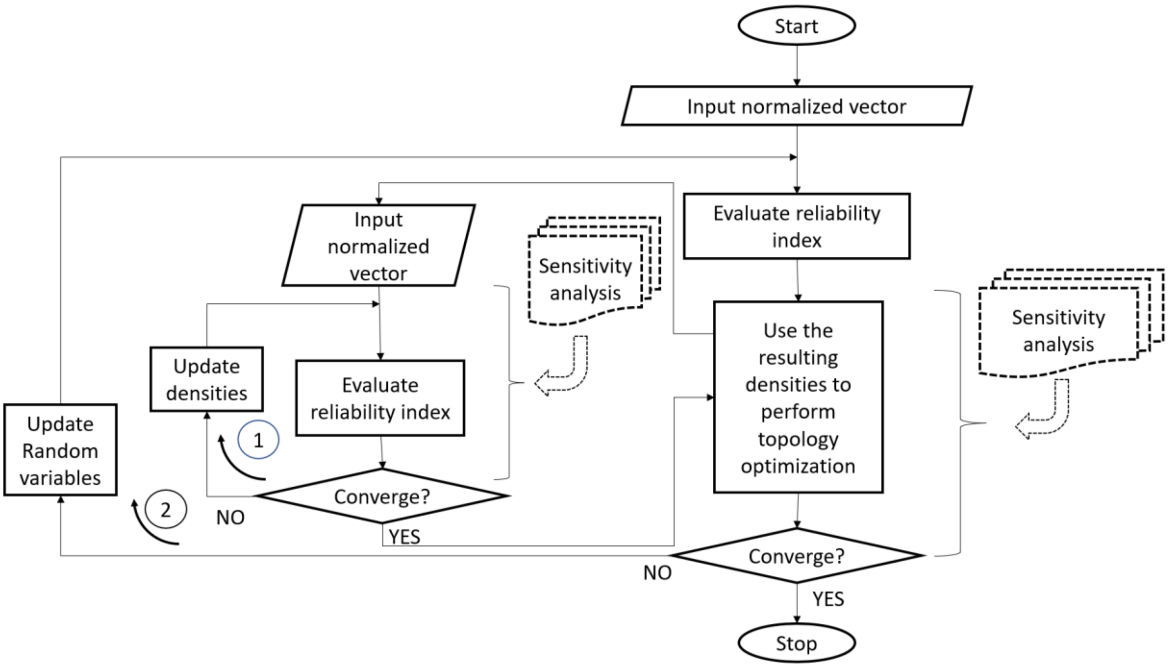


**Figure 4.** *Generalized flowchart for nested loop methods*

Several researchers developed their RBTO model considering this strategy. Kang and Luo (2009), Wang et al. (2018a) and Wang et al. (2018b) established a NRBTO model considering these nested loops. Recently, Pham and Hoyle (2021) dealt with robust reliability-based topology optimized designs considering this double loops.

### 2.2.2. *Mixed methods:*

Other methods have been developed to meet improve the robustness and/or efficiency. In general, the basic idea of these methods is to solve the probabilistic problem in a separate way. So, the probabilistic problem solution is carried out considering several ways which are presented in this section.

#### 2.2.2.1. Simulation-based methods

In certain RBTO methods, the reliability analysis process is performed by an iterative simulation method such as Monte-Carlo simulation. Figure 5 shows a generalized flowchart of the simulation-based methods where an additional bock is added in red colour to show that the simulation is performed before the topology optimization process. This simulation is an iterative procedure until the number of iterations is met. There is no optimization process at this first stage which leads to evaluate the reliability index. However, the topology optimization is performed using regular optimization method where the sensitivity analysis module can be needed.

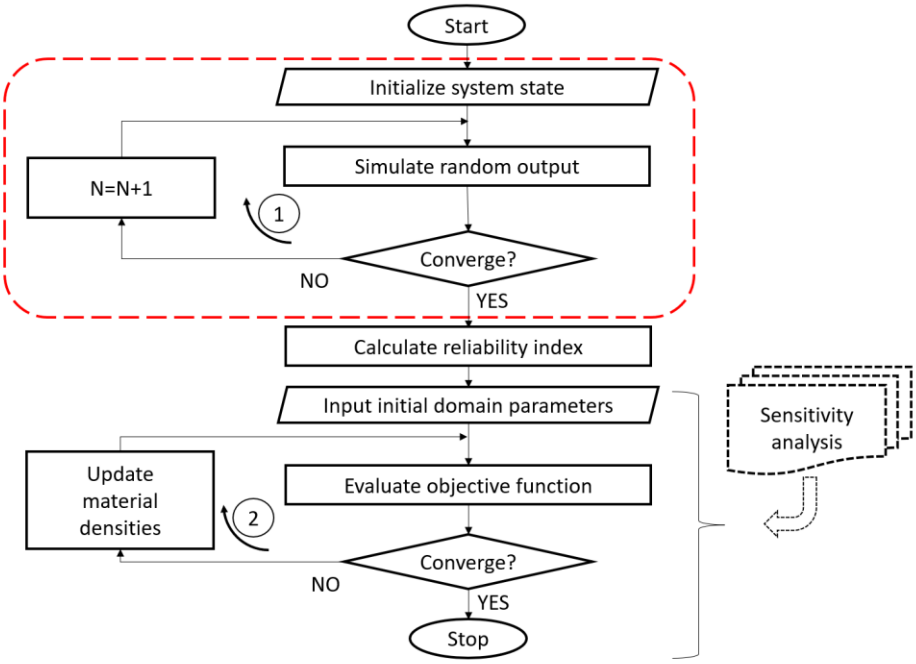


**Figure 5.** *Generalized flowchart for simulation-based methods*

In Bobby et al. (2017), Monte Carlo simulation was integrated into the RBTO process. It was called a simulation based RBTO method. The probabilistic analysis is decoupled from the optimization loop and use Monte Carlo simulations to deal with the probabilistic part. The proposed RBTO algorithm was applied to two structures undergoing to non-stationary stochastic earthquake excitation. Here, it is very important for this kind of applications to carry out the accuracy during the design process. Despite Monte Carlo simulation can be considered as an expensive technique, the obtained results can be considered accurate.

### 2.2.2.2. Approximation-based methods

In certain RBTO methods, the reliability analysis is performed by an approximation method such as Response Surface Method. Figure 6 shows a generalized flowchart of the approximation-based methods where an additional block is added in red colour to show that the approximation process is carried out before the topology optimization process. In this approximation, the studied domain is divided into set of experiment points and the different functions are calculated at these points. There is a special optimization procedure at this stage that leads to evaluate the different responses. Next, the topology optimization is carried out using regular optimization method where the sensitivity analysis module can be utilized.

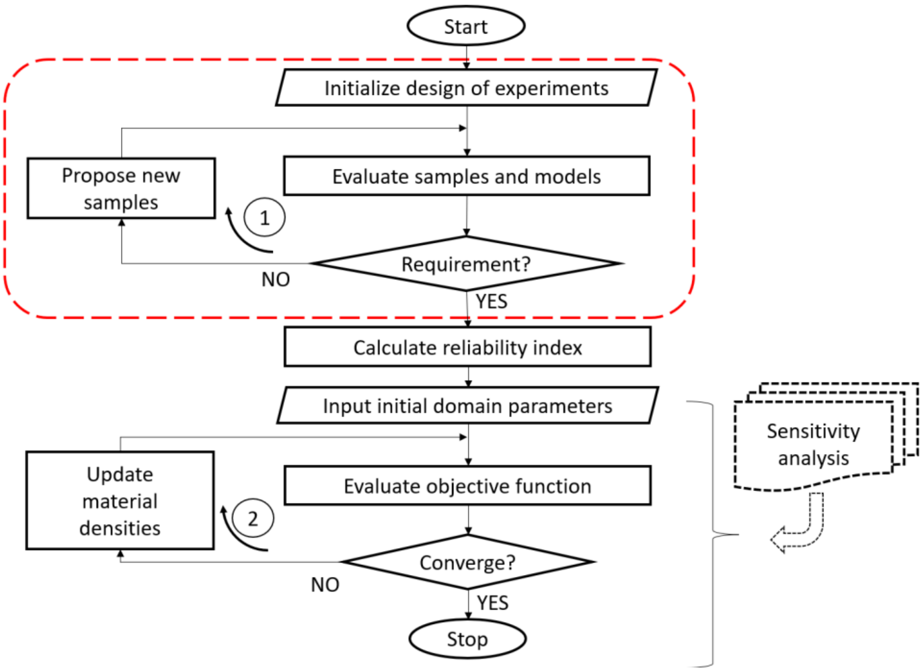


**Figure 6.** *Generalized flowchart for approximation-based methods*

In this context, Yoo et al. (2011) elaborated a successive standard response surface method and Zhao et al. (2015) next developed a stochastic response surface method.

### 2.2.2.3. Optimality condition-based methods

Figure 7 shows a generalized flowchart for optimality condition-based method where two separate topology optimization loops are used. A block in red colour can be seen for the use of the analytical formulations. This kind of methods doesn't lead only to a high reduction of computational time consumption, but also to obtain at least a local optimum when satisfying the optimality conditions. Therefore, this kind of methods cannot be considered as general or robust methods since they can be used for specific applications and there is no guarantee to get a global optimum. They can only meet a couple of criteria: efficiency and the capacity.

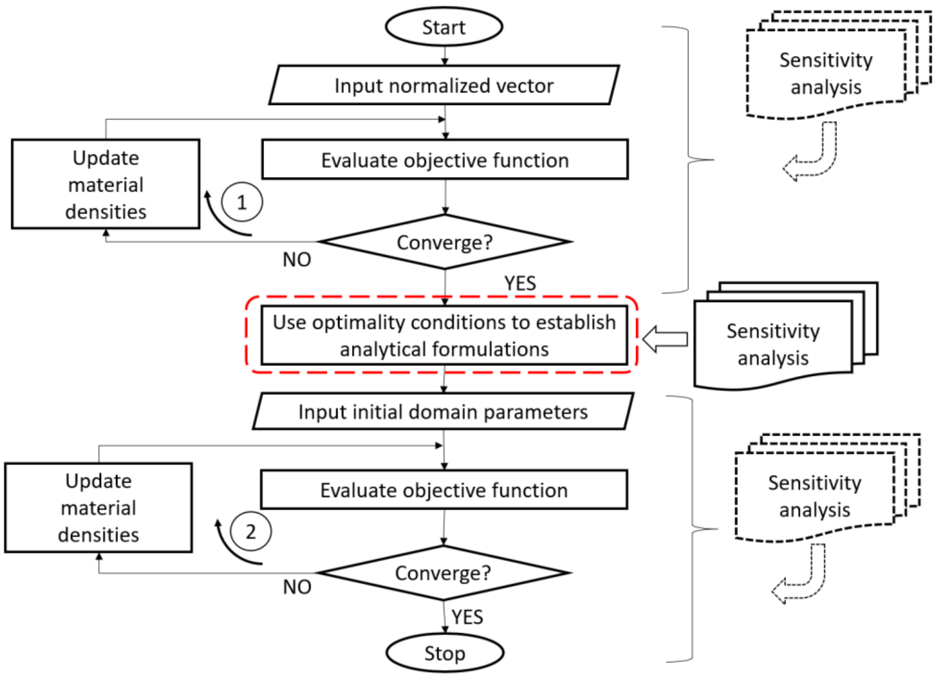


**Figure 7.** *Generalized flowchart for optimality condition-based methods*

In this context, Kharmanda and Antypas (2019a) started to use the OSF method which is based on optimality condition developments to elaborate IOSF one. Recently, Meng et al. (2020) developed a hybrid RBTO method which was also established on Karush–Kuhn–Tucker optimality condition.

#### 2.2.2.4. Decision making methods

Decision making methods can be differentiated in their general flowchart according to the used decision-making techniques such as neural network, fuzzy logic ... The objective of this external technique is to evaluate the probabilistic constraints. To implement the neural networks in the RBTO model, several flowcharts can be found in literature (see Patel and Choi (2012)). Furthermore, the fuzzy logic strategy can be modelled in several ways (see Yin et al. (2018)). Figure 8 shows a generalized flowchart for decision making methods using neural networks, while Figure 9 shows a generalized flowchart for decision making methods using fuzzy logic strategy.

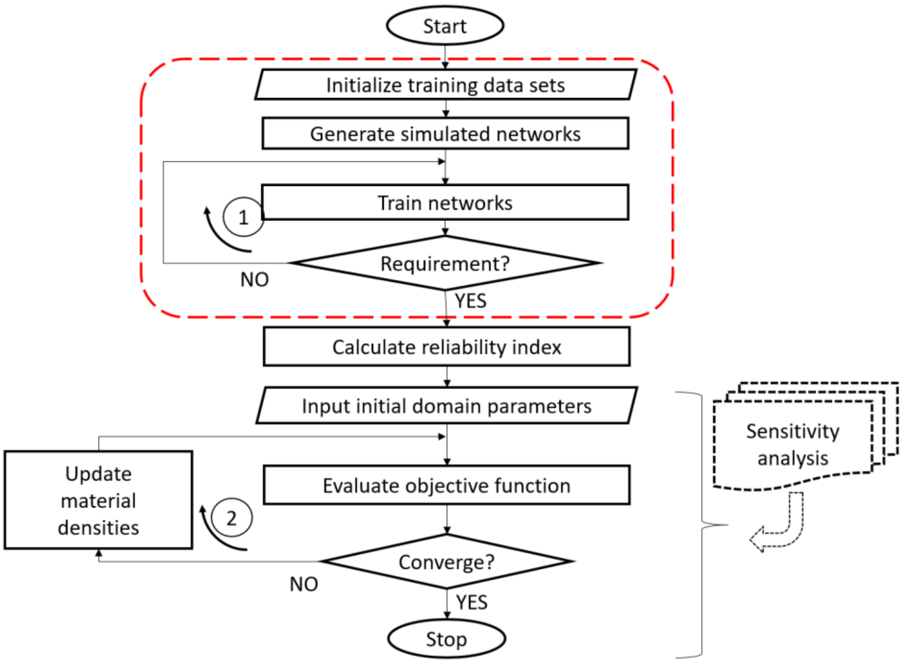


**Figure 8.** *Generalized flowchart for decision-making methods (neural networks)*

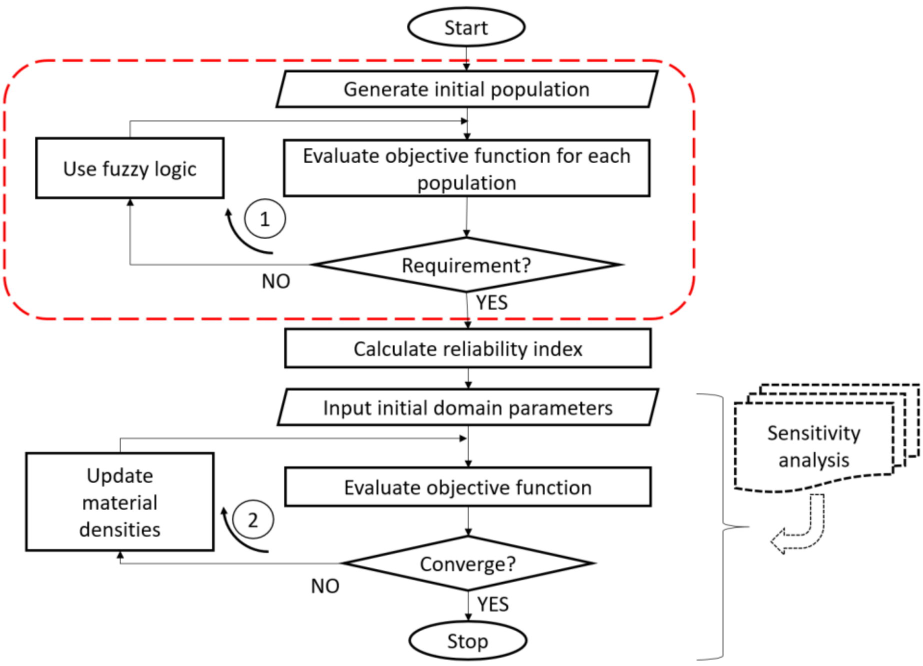


**Figure 9.** *Generalized flowchart for decision-making methods (fuzzy logic)*

In this context, Patel and Choi (2012) developed a classification approach for RBTO model using probabilistic neural networks and Yin et al. (2018) next used a fuzzy method to solve RBTO problems.

## 3. Discussion and research gaps

During the last two decades, several critical comments led to a significant change in the trends of the RBTO model. This can be noted when extending the RBTO developments to several areas. According to the RBTO formulation classification, the majority of the different RBTO developments in literature can be divided into two main groups. The first group called developments from a topology optimization standpoint, which leads to various layouts with decreasing rigidity (increasing compliance) levels. Despite, the reliability constraints were integrated into the DTO process in a simple way (Kharmanda and Olhoff (2001)), several problems appeared later. Some researchers consider that there is no physical meaning when representing the limit state function by a prescribed volume constraint. Furthermore, when considering the uncertainties of the geometry, this may lead to manufacturing errors (Liu et al. (2015)). Nevertheless, the second group being called developments from a reliability analysis standpoint, often provides the same layouts with increasing rigidity (or decreasing compliance) levels. The single disadvantage of these methods is to provide the same layouts with dissimilar thickness. Some researchers consider that this conclusion does not represent any value since a detailed design stage is required to control the structural rigidity. In this review paper, four criteria are considered for comparison: generality, robustness, capacity, and efficiency. Some methods can be general to many problems, while others such as optimality conditions-based method, can be used for specific applications. The robustness of the used method is necessary in certain industry such as aviation industry. Here, the accuracy of results plays an important role to select the most suitable method. Despite, the simulation-based methods such as Monte Carlo method, are highly time consuming, they are needed to obtain reliable results. Next, the capacity of the method in solving problem with a big number of variables and constraints can be considered as an important criterion for certain applications. For large-scale problems, nested loop methods may suffer from a weak convergence stability and a high computing time consumption. In general, the First-Order Reliability Method (FORM) is often used to efficiently obtain an estimate of the reliability of a system here. This approach treats the reliability analysis as a nested optimization problem, where the objective is to find the most probable point (MPP) by minimizing the distance between the failure surface and the origin of a normalized random space. Numerical gradient calculation of the solution of this nested problem necessitates an additional solution of the FORM problem for each design variable. In addition, there are two common FORM-based approaches are RIA and its inverse, the more efficient PMA. Kim et al. (2006) formulated the RBTO problems based on the FORM, where both approaches were compared with each other. Their results clearly indicated that the PMA had a better convergence and efficiency than the RIA (Zhao et al. (2016)). Therefore, this approach quickly becomes computationally intractable for large scale problems including the RBTO model. So, it is difficult to use these kinds of methods for large-scale applications. However, in order to improve the computational efficiency and accuracy in future research, the reliability analysis could be combined with sequential approximation and linear approximation strategies to cope with the nested optimization problems (Yin et al. 2021). Finally, for the efficiency criterion, several RBTO methods suffer from the computational times, there is a need to reduce the computational time consumption. For example, an alternative analytic approach to the analysis and sensitivity of nested optima derived from the Lagrange Multiplier Theorem is explored in Clark (2017). This approach leads to a system of nonlinear equations for the MPP analysis for any given set of design variables. Taking the derivative of these equations with respect to a design variable gives a linear system of equations in terms of the implicit sensitivities of the MPP to the design variable where the coefficients of the linear equations depend only on the current MPP. By solving this system, these sensitivities can be obtained without requiring addition solutions of the FORM problem. The proposed approach is demonstrated through several RBDO and RBTO problems. In addition, to replace a calculation part by other technique can be very helpful to increase the robustness and/or the efficiency. In addition, the decoupled loop and optimality conditions-based methods seem to be the most efficient methods. They do not necessitate a high time consumption. Regarding the different criteria, the decoupled loop methods may meet three criteria such as efficiency, generality, capacity.

## 4. Future research directions

The topology of the optimum solution is usually different for an RBTO problem, where the randomness in loads and geometry are incorporated in the problem definition, compared to a deterministic topology optimization (DTO) problem, where the uncertainties are not considered in the problem solution. This is the basic idea behind incorporating reliability constraints early in the design process, i.e. even before design optimization, since the resulting topology of a structure from an RBTO exhibits a better volume/reliability ratio compared to a DTO solution. In other words, because the resulting topologies obtained from RBTO and DTO can be different, RBTO is able to accomplish the same reliability levels with smaller structural volume compared with a two-step process in which DTO is performed first and a reliability-based design optimization (RBDO) is performed in the second step on the optimum topology obtained from DTO. This fact is presented in Kharmanda et al. (2004), where the incorporation of reliability constraints early on is shown to be advantageous in terms of finding a lighter and reliable structure as a final design compared to including the reliability constraints at the design optimization stage. This has led to several research efforts focused on developing efficient methods to perform RBTO (Kanakasabai and Dhingra (2014)). In addition, when dealing with recent developments from reliability analysis standpoints, the use of NRBTO strategies shows that the results may lead to topological layouts that are significantly different from the traditional deterministic optimization results (Ma and Wang 2021). Many efforts are needed to avoid the use of nested loop methods, especially when considering the reliability analysis standpoint without sacrificing solution accuracy. For example, Pham and Hoyle (2021) found that only the robust reliability-based design approach can identify the optimal topology considering the influence of material property uncertainty in both the objective and the constraint functions. There is a need to integrate multi-source uncertainties in practical applications (for example, dynamic responses) which have become a research hotspot for future development of intelligent design technologies as well as multi-functional and multi-scale design conditions (Wang et al. 2021b). One of the most important applications is the composite material domain. Composite materials process several advantages such as high strength, high rigidity, light weight, design flexibility, etc. The use of the RBTO model in composite materials is still in the initial stage. Several issues can be found in this area and need to be solved such as the optimization policies under stiffness and strength integrated failure modes, the hybrid reliability judgments under aleatory and epistemic uncertainties, the difficulties in global optimization and fast convergence for complex laminated structures, etc (Wang et al. 2021c). Furthermore, the RBTO developments extended recently to microstructure applications where the components become smaller and smaller. So, the computational time plays and important role to select the best method. Here, the microscale uncertainties of material properties and microstructures can affect the overall structure performances. So, classical deterministic topology optimization in which material properties and loadings are assumed to be deterministic would be insufficient (Gao and Liu 2021). When dealing with several disciplines, the RBTO model leads to several layouts taking several objectives into account. In the work of Pérez-Rúa et al. (2021), different objectives (economy, operation, energy …) were treated simultaneously and several strategies were incorporated in their algorithm in order to solve large real-world problems. Thus, there is a need to develop efficient tools to integrate the RBTO model into multidisciplinary optimization strategies.

After two decades of developments, the different efforts for both of standpoints start to converge to improve the resulting reliability-based topology layouts. So, there is a need to develop new strategies to combine between the two standpoints to produce several layouts with increasing rigidity levels in a reasonable computing time consumption. Considering that the RBTO leads to several topologies, it can be integrated to additive manufacturing to solve some issues such as support structures, optimum material distribution with the object of industrialize this technology. In the context of additive manufacturing, it will be significant considering material property uncertainty. Here, the additive process can lead to significant material property uncertainty due to temperature gradients or other sources of variation. Furthermore, when considering topology optimization as a helpful tool for additive manufacturing, another important characteristic of the material property is non-linearity (Pham and Hoyle (2021)). The recent RBTO developments at microstructural levels can be very helpful to solve several issues in micro additive manufacturing. Figure 10 shows a small pen cup with a small thickness where there is no need to use support structures. When the dimensions become smaller and smaller, micro/nanoscale wires or yarns may appear in the case where overhanging features exist in the additive manufactured structures (such as the diamond-shaped holes in Figure 10). In fact, these micro/nanoscale wires or yarns prevent the filament to go down and the additive manufacturing process continues in a safe way. So, this microstructural additive manufacturing level in certain cases allows to reduce probability of using support structures when performing the resulting topologies which can be very useful in microfabrication fields such as biomedicine, electromechanics ...


**Figure 10.** *Micro additive manufactured structure without support structures (Picture belongs to 3D printing 4U (UG), see https://3d-printing-4u.com/)*

Finally, the selection of the suitable method depends on the requirements (ex. time consumption) and the application itself. For example, the dynamic reliability-based topology optimization approach is more complicated to be applied to static problems (Wang et al. 2021b). The mixed methods can be considered as derivatives from loop methods since one loop is replaced by another technique (simulation, approximation ...) which can be itself as an iterative loop. The available tools are considered as restrictions (some limitations to apply the different methods). So, there a need to develop new strategies and techniques to balance between the different criteria.


## Acknowledgement:

The authors would like to thank Dr A. Shokry from Nile University and Dr S. Gowid from Qatar University for their valuable comments during the elaboration of this review.


## References:


Kharmanda, G.; Olhoff, N. (2001): Reliability-Based Topology Optimization, Report N°: 110, Institute of Mechanical Engineering, Aalborg University, Denmark, December 2001.

Kharmanda, G.; Olhoff, N. (2002): Reliability-Based Topology Optimization as a New Strategy to Generate Different Topologies /In: The Nordic Seminar in Computational Methods, (eds. E. Lund, N. Olhoff and J. Stegmsen), Aalborg University, Denmark, October. 2002, 11-14.

Bae, K.; Wang, S. (2002): Reliability-based topology optimization /Proceedings of 9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization. AIAA, 2002, pp. 2002-5542.

Kang, J.; Kim, C.; Wang, S. (2004): Reliability-based topology optimization for electromagnetic system, COMPEL - The international journal for computation and mathematics in electrical and electronic engineering, 23(3), pp. 715-723. https://doi.org/10.1108/03321640410540647

Kharmanda, G.; Olhoff, N.; Mohamed, A.; Lemaire, M. (2004): Reliability-Based Topology Optimization /Journal of Structural and Multidisciplinary Optimization. 2004, 26, pp. 295–307.

Jung, H.S.; Cho, S. (2004): Reliability-based topology optimization of geometrically nonlinear structures with loading and material uncertainties, Finite Elements in Analysis and Design, 43, 2004, pp. 311–331.

Moon, H.; Kim, C.; Wang, S. (2004): Reliability-Based Topology Optimization of Thermal Systems Considering Convection Heat Transfer, 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, 30 August - 01 September 2004, Albany, New York. https://doi.org/10.2514/6.2004-4410

Patel, N.M.; Agarwal, H.; Tovar, A.; Renaud, J. (2005): Reliability based topology optimization using the hybrid cellular automaton method /In 1st AIAA Multidisciplinary Design Optimization Specialist Conference, Austin, Texas. 2005, pp. 18-21.

Patel, N.M.; Agarwal, H.; Tovar, A.; Renaud, J. (2005): Reliability Based Topology Optimization using a Hybrid Cellular Automaton Algorithm, 6th World Congresses of Structural and Multidisciplinary Optimization, Rio de Janeiro, 30 May - 03 June 2005, Brazil.

Kim, C.; Wang, S.; Kang, E.; Lee, K. (2006a): New Design Process for Reliability-Based Topology Optimization of a Laser Scanned Model, Mechanics Based Design of Structures and Machines, 34 (3), 2006, pp. 325-347.

Kim, C.; Wang, S.; Bae, K.; Moon, H. (2006b): Reliability-based topology optimization with uncertainties. J Mech Sci Technol, 20(4), pp. 494–504

Wang, S.; Moon, H.; Kim, C.; Kang, J.; Choi, K.K. (2006): Reliability-based topology optimization (RBTO), In IUTAM Symposium on Topological Design Optimization of Structures, Machines and Materials, pp. 493-504.

Mozumder, C. K.; Renaud, J.E. (2006a): An Investigation of reliability-based topology optimization, 20th Aerospace and Mechanical Engineering Graduate Student Conference, Notre Dame, Indiana, USA, 19 October 2006.

Mozumder, C.; Patel, N.; Tillotson, D.; Renaud, J.; Tovar, A. (2006b): An Investigation of Reliability-Based Topology Optimization, 11th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, 06 - 08 September 2006, Portsmouth, Virginia. https://doi.org/10.2514/6.2006-7058

Kogiso, N.; Mogami, K.; Nishiwaki, S.; Izui, K.; Yoshimura, M. (2006): Reliability-Based Topology Optimization of Frame Structure with Multiple Modes, 47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, 01 - 04 May 2006, Newport, Rhode Island. https://doi.org/10.2514/6.2006-2002

Wang, S.; Moon, H.; Kim, C.; Kang, J.; Choi, K.K. (2006): RELIABILITY-BASED TOPOLOGY OPTIMIZATION (RBTO), Martin P. Bendsøe et al. (eds), IUTAM Symposium on Topological Design Optimization of Structures, Machines and Materials: Status and Perspectives, pp. 493–504. © 2006 Springer. Printed in the Netherlands.

Kim, C.; Wang, S.; Hwang, I.; Lee, J. (2007a): Application of Reliability-Based Topology Optimization for Microelectromechanical Systems, AIAA Journal, 45(12), 2007.

Kim, J. Park, W.-G. Lee, J.-S. Yu, S.-Y. Han (2007b): Reliability-based topology optimization based on evolutionary structural optimization, Int. J. Mech. Syst. Sci. Eng., 1 (2007), pp. 135–9.

Kharmanda, G.; Lambert, S.; Kourdi, N.; Daboul, A.; El-Hami, A. (2007a): Reliability-based topology optimization for different engineering applications, International Journal of CAD/CAM .2007, pp. 61-69.

Kharmanda, G.; Kourdi, N.; Daboul, A.; Lambert S.; El Hami, A. (2007b): Reliability-based topology optimization as efficient tool for structural designers, 2ème Congrès International Francophone de Mécanique Avancée, CIFMA02, 2nd International Francophone Congress of Advanced Mechanics, IFCAM02, 14-16 May 2007, Aleppo, Syria.

Zhang, X.; Ouyang, G. (2008): A level set method for reliability-based topology optimization of compliant mechanisms, Science in China Series E: Technological Sciences, 51(4), pp. 443-455

Kogiso, N., Hirano, Y.; Nishiwaki, S.; Izui, K.; Yoshimura, M.; Min, S. (2009): Reliability-based topology optimization for frame structures considering multiple performance criteria using SLSV method with improvement of convergence properties. Transactions of Japan Society of Mechanical Engineers, Part C, 75(751), pp 719-726.

Kang, Z.; Luo, Y. (2009): Non-probabilistic reliability-based topology optimization of geometrically nonlinear structures using convex models, Computer Methods in Applied Mechanics and Engineering, 198 (41–44), 1 September 2009, pp. 3228-3238.

Silva, M.; Tortorelli, D.A.; Norato, J.A.; Ha, C.; Bae, H.R. (2010): Component and system reliability-based topology optimization using a single-loop method, Structural and Multidisciplinary Optimization. (2010) 41, pp. 87–106. DOI 10.1007/s00158-009-0401-5

Kogiso N, Hirano Y, Nishiwaki S, Izui K, Yoshimura M, Min S (2010): Reliability-based topology optimization offrame structures for multiple criteria using SLSV method, Journal of Computational Science and Technology, 4, pp. 172-184.

Patel, J. (2010): Optimal Design of Meso-structured Materials under Uncertainty: Reliability Based Topology Optimization (RBTO) based method for design of meso-structures, LAP LAMBERT Academic Publishing (December 3, 2010), 112 pages, ISBN-10: 9783843379793.

Eom, Y.S.; Yoo, K.S.; Park, J.Y.; Han, S.Y. (2011): Reliability-based topology optimization using a standard response surface method for three-dimensional structures, Structural and Multidisciplinary Optimization. 2011, 43 (2), pp. 287–295.

Olyaie, M.S.; Razfar, M.R.; Kansa, E.J. (2011): Reliability Based Topology Optimization of a Linear Piezoelectric Micromotor Using the Cell-Based Smoothed Finite Element Method, CMES, 75(1), pp.43-87, 2011.

Mashayekhi, M.; Salajegheh, J.; Fadaee, M.J.; Salajegheh, E. (2011): A two-stage SIMP-ACO method for reliability-based topology optimization of double layer grids, Int. J. Optim. Civil Eng., 2011; 4, pp. 521-542.

Yoo, K-S.; Eom, Y-S.; Park, J-Y.; Im, M-G.; Han, S-Y. (2011): Reliability-based topology optimization using successive standard response surface method, Finite Elements in Analysis and Design, 47 (7), 2011, pp. 843-849.

Nguyen, T.H.; Song, J.; Paulino, G.H. (2011): Single-loop system reliability-based topology optimization considering statistical dependence between limit-states, Structural and Multidisciplinary Optimization, 44, pp. 593–611.

Cho, K.-H.; Park, J.-Y.; Ryu, S.-P.; Park, J.-Y.; Han, S.-Y. (2011): Reliability-based topology optimization based on bidirectional evolutionary structural optimization using multi-objective sensitivity numbers, Int. J. Auto. Tech. 12, (2011), pp. 849–56.

Li, J.; Zhang, Y.; Yu, C.; Guo, J. (2011): The reliability-based topology optimization design by stochastic perturbation, 2011 IEEE International Conference on Computer Science and Automation Engineering, Shanghai, 10-12 June 2011. DOI: 10.1109/CSAE.2011.5952820

Patel, J.; Choi, S.K. (2012): Classification approach for reliability-based topology optimization using probabilistic neural networks. Structural and Multidisciplinary Optimization. 2012, 45 (4), pp. 529–543.

Mashayekhi, M.; Salajegheh, E.; Salajegheh, J.; Fadaee, M.J. (2012): Reliability-based topology optimization of double layer grids using a two-stage optimization method, Structural and Multidisciplinary Optimization, 45, pp. 815–833.

Xu, B. (2013): Dynamic non-probabilistic reliability-based topology optimization of truss with uncertain-but-bounded parameters, Journal of Vibration and Control, 21(12), pp. 2484-2496.

Olyaie, M.S.; Razfar, M.R.; Wang, S. (2013): Reliability-based topology optimization design of a linear piezoelectric micromotor using an optimum finite element method, Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability, 227(6), pp. 586-598.

Jalalpour, M.; Guest, J.; Igusa, T. (2013): Reliability-based topology optimization of trusses with stochastic stiffness", Structural Safety, 43, pp. 41-49, 2013.

Torii, A.J.; Novotny, A.A. (2013): Reliability based topology optimization using topological derivatives and sequential optimization and reliability assessment, Proceedings of the XXXIV Iberian Latin-American Congress on Computational Methods in Engineering (CILAMCE 2013), Z.J.G.N Del Prado (Editor), ABMEC, Pirenopolis, GO, Brazil, November 10-13, 2013.

Zhao Q, Chen X, Lin Y (2013) Reliability-based topology optimization of control arm of suspension for lightweight design. In: Proceedings of the FISITA 2012 World Automotive Congress, pp. 1129-1138.

El-Hami, A.; Radi, B. (2013): Reliability-Based Topology Optimization, Chapter 8, Wiley. https://doi.org/10.1002/9781118711903.ch8

Luo, Y.; Zhou, M.; Wang, M.Y.; Deng, Z. (2014): Reliability based topology optimization for continuum structures with local failure constraints, Computers & Structures, 143, September 2014, pp. 73-84.

Kanakasabai, P.; Dhingra, A.K. (2014): An efficient approach for reliability-based topology optimization, Engineering Optimization, 2014. http://dx.doi.org/10.1080/0305215X.2014.992890

Zhao, Q.; Chen, X.; Ma, Z-D.; Lin, Y. (2015): Reliability-Based Topology Optimization Using Stochastic Response Surface Method with Sparse Grid Design, Mathematical Problems in Engineering, Article ID 487686, 13 pages. http://dx.doi.org/10.1155/2015/487686

Jalalpour, M.; Tootkaboni, M. (2016): An efficient approach to reliability-based topology optimization for continua under material uncertainty. Journal of Structural and Multidisciplinary Optimization. 2016, 53, pp. 759-772.

Liu, J.; Wen, G.; Zuo, H.Z.; Qing, Q. (2016a): A simple reliability-based topology optimization approach for continuum structures using a topology description function, Engineering Optimization, 48 (7), pp. 1182-1201, 2016

Kanakasabai, P.; Dhingra, A.K. (2016): An efficient approach for reliability-based topology optimization, Engineering Optimization, 48(1), pp. 1-15, 2016.

Liu, K.; Paulino, G.H.; Gardoni, P. (2016b): Reliability-based topology optimization using a new method for sensitivity approximation - application to ground structures, Structural and Multidisciplinary Optimization, 54, pp. 553–571 (2016).

López, C.; Baldomir, A.; Hernández, S. (2016): Deterministic versus reliability-based topology optimization of aeronautical structures, Structural and Multidisciplinary Optimization, 53 (4), 2016, pp. 907-921.

Zhao, Q.; Chen, X.; Ma, Z.; Yi Lin, Y. (2016): A Comparison of Deterministic, Reliability-Based Topology Optimization under Uncertainties, Acta Mechanica Solida Sinica, 29(1), February, 2016, ISSN 0894-9166.

Wang, L.; Liu, D.; Yang, Y.; Wang, X.; Qiu, Z. (2017): A novel method of non-probabilistic reliability-based topology optimization corresponding to continuum structures with unknown but bounded uncertainties. Computer Methods in Applied Mechanics and Engineering, 2017, 326, pp. 573-595.

Bobby, S.; Suksuwan, A.; Spence, S.M.J.; Kareem, A. (2017): Reliability-based topology optimization of uncertain building systems subject to stochastic excitation, Structural Safety, 66, 2017, pp. 1-16.

Lee, J.K.; Lee, J.W. (2017): Reliability-based topology optimization for optimal muffler design considering uncertainty, INTER-NOISE and NOISE-CON Congress and Conference Proceedings, InterNoise17, Hong Kong CHINA, pages 1004-1995, pp. 1854-1858(5), Institute of Noise Control Engineering, December 7, 2017.

Clark, P.R. (2017): Reliability-Based Topology Optimization with Analytic Sensitivities, M.Sc. Dissertation, June 19, 2017, Virginia, Virginia Polytechnic Institute and State University.

Kharmanda, G.; El-Hami, A. (2017): Reliability-based topology optimization model, Chapter N. 5, in book 'Biomechanics: Optimization, Uncertainties and Reliability', ISTE Ltd and John Wiley & Sons, Inc, 13 Feb. 2017.

Kang, Z.; Liu, P. (2018): Reliability-based topology optimization against geometric imperfections with random threshold model, Numerical Methods in Engineering, 115(1), pp. 99-116.

Da Silva, G.A.; Beck, A.T. (2018): Reliability-based topology optimization of continuum structures subject to local stress constraints, Structural and Multidisciplinary Optimization, 57, pp. 2339–2355 (2018).

Yin, H.; Yu, D.; Xia, B. (2018): Reliability-based topology optimization for structures using fuzzy set model, Computer Methods in Applied Mechanics and Engineering, 333, 2018, pp. 197-217.

Wang, L.; Liang, J.; Wu, D. (2018): A non-probabilistic reliability-based topology optimization (NRBTO) method of continuum structures with convex uncertainties, Structural and Multidisciplinary Optimization volume, 58, pp. 2601–2620 (2018).

Zheng, J.; Luo, Z.; Jiang, C.; Ni, B.; Wu, J. (2018a): Non-probabilistic reliability-based topology optimization with multidimensional parallelepiped convex model, Structural and Multidisciplinary Optimization, 57, pp. 2205–2221 (2018).

dos Santos, R.B.; Torii, A.J.; Novotny, A.A. (2018): Reliability-based topology optimization of structures under stress constraints, Numerical Methods in Engineering, 114(6), pp. 660-674.

López, C.; Baldomir, A.; Hernández, S. (2018): The relevance of reliability-based topology optimization in early design stages of aircraft structures, Structural and Multidisciplinary Optimization, 57, pp. 417–439 (2018).

Papadimitriou, D.I.; Mourelatos, Z.P. (2018): Reliability-Based Topology Optimization Using Mean-Value Second-Order Saddle point Approximation, J. Mech. Des. Mar 2018, 140(3): 031403 (11 pages)

Wang, L.; Cai, Y.; Liu, D. (2018b): Multiscale reliability-based topology optimization methodology for truss-like microstructures with unknown-but-bounded uncertainties, Computer Methods in Applied Mechanics and Engineering, 339, 2018, pp. 358-388.

Li, Z.; Chen, J.; Shi, L.; Bian, H. (2018): Reliability-based Topology Optimization of Continuum Structure, Proceedings of ICIMM 2018, ISBN: 978-989-758-345-2, Proceedings of the 2nd International Conference on Intelligent Manufacturing and Materials, June 15-16, 2018, in Guangzhou, China

Kharmanda, G.; Antypas, I.; Dyachenko, A. (2019a): Inverse Optimum Safety Factor Method for Reliability-Based Topology Optimization Applied to Free Vibrated Structures, Journal of Engineering Technologies and Systems. 2019, 29 (1), pp. 8-19.

Kharmanda, G.; Antypas, I.; Dyachenko, A. (2019b): The Effect of Reliability Index Values on Resulting Reliability-Based Topology Optimization Configurations: Numerical Validation by Shape Optimization, Journal of Engineering Technologies and Systems. 2019, 29 (3), pp. 322-344.

Wang, L., Liang, J., Liu, D., & Chen, W. (2019a). A novel reliability-based topology optimization framework for the concurrent design of solid and truss-like material structures with unknown but bounded uncertainties. International Journal for Numerical Methods in Engineering, 119(4), pp. 239-260.

Wang, L., Xia, H., Yang, Y., Cai, Y. and Qiu, Z. (2019b), "A novel approach of reliability-based topology optimization for continuum structures under interval uncertainties", Rapid Prototyping Journal, 25(9), pp. 1455-1474. https://doi.org/10.1108/RPJ-08-2017-0163

Chun, J.; Paulino, G.H.; Song, J. (2019): Reliability-based topology optimization by ground structure method employing a discrete filtering technique, Structural and Multidisciplinary Optimization (2019), 60, pp. 1035–1058. https://doi.org/10.1007/s00158-019-02255-1

Wang, L.; Liu, D.; Yang, Y.; Hu, J. (2019c): Novel methodology of Non-probabilistic Reliability-based Topology Optimization (NRBTO) for multi-material layout design via interval and convex mixed uncertainties, Computer Methods in Applied Mechanics and Engineering, 346, 2019, pp. 550-573.

Wang, L.; Xia, H.; Zhang, X.; Lv, Z. (2019d): Non-probabilistic reliability-based topology optimization of continuum structures considering local stiffness and strength failure, Computer Methods in Applied Mechanics and Engineering, 346, 2019, pp. 788-809.

Chun, J.; Paulino, G.H.; Song, J. (2019): Reliability-based topology optimization by ground structure method employing a discrete filtering technique, Structural and Multidisciplinary Optimization, 60, pp. 1035–1058 (2019).

Vishwanathan, A.; Vio, G.A. (2019): Efficient quantification of material uncertainties in reliability-based topology optimization using random matrices, Computer Methods in Applied Mechanics and Engineering, 351, 2019, pp. 548-570.

Wang, L.; Liu, Y.; Yang, Y. (2019e): Truss layout design under nonprobabilistic reliability-based topology optimization framework with interval uncertainties, Numerical Methods in Engineering, 119(12), 2019, pp. 1307-1324.

Wang, L.; Liang, J.; Chen, W.; Qiu, Z. (2019f): A nonprobabilistic reliability–based topology optimization method of compliant mechanisms with interval uncertainties, Numerical Methods in Engineering, 119(13), 2019, pp. 1419-1438.

Kharmanda G.; Shokry A.; Gowid S.; El-Hami A. (2019): Comparison of two points of view when developing reliability-based topology optimization model: Validation on fatigue damage analysis, Journal of Uncertainties and Reliability of Multiphysical Systems, 19-3 (1), December 2019.

Sato, Y.; Izui, K.; Yamada, T.; Nishiwaki, S.; Ito, M.; Kogiso, N. (2019): Reliability-based topology optimization under shape uncertainty modeled in Eulerian description, Structural and Multidisciplinary Optimization, 75. https://doi.org/10.1007/s00158-018-2051-y

Chun, J.; Park, D.; Lee, J. (2019): TOPO-Joint: Reliability-based topology optimization for 3D-printed building joints, 13th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP13, Seoul, South Korea, May 26-30, 2019.

Kharmanda, G.; El-Hami, A.; Antypas, I. (2019c): Inverse optimum safety factor method as an effective tool for reliability-based topology optimization: Application on static and dynamic cases, Fifth International Conference on Soft Computing & Optimisation in Civil, Structural and Environmental Engineering, Riva del Garda, near Lake Garda, Italy, 16-19 September 2019.

Meng, Z.; Pang, Y.; Pu, Y.; Wang, X. (2020): New hybrid reliability-based topology optimization method combining fuzzy and probabilistic models for handling epistemic and aleatory uncertainties, Computer Methods in Applied Mechanics and Engineering. 2020, 363, Article 112886.

Sleesongsom, S.; Bureerat, S. (2020): Multi-objective reliability-based topology optimization of structures using a fuzzy set model, Journal of Mechanical Science and Technology, 34, pp. 3973–3980 (2020).

Kharmanda G.; Antypas I.; Dyachenko A. (2020): Reliability-Based Topology Optimization Using Two Alternative Inverse Optimum Safety Factor Approaches: Application to Bridge Structures, Journal of Engineering Technologies and Systems, 30(3), pp. 498-511. DOI: 10.15507/2658-4123.030.202003.498-511

Kharmanda, G.; Antypas, I. (2020a): Reliability-based topology optimization as effective strategy for additive manufacturing: Influence study of geometry uncertainty on resulting layouts, Journal of Physics Conference Series, 1679, November 2020. DOI: 10.1088/1742-6596/1679/4/042052

Kharmanda G.; Gowid S.; Shokry A. (2020a): Reliability-Based Topology Optimization using Inverse Optimum Safety Factor Approaches, Alexandria Engineering Journal, 59(6), December 2020, pp. 4577-4592.

Kharmanda, G.; Antypas, I. (2020b): Reliability-Based Topology Optimization as Effective Strategy for Additive Manufacturing: Influence study of geometry uncertainty on resulting layouts, II International Scientific Conference on Applied Physics, Information Technologies and Engineering - APITECH-II-2020, 25 Sep. - 4 Oct. 2020, Krasnoyarsk, Russia.

Kharmanda, G.; Antypas, I.; Dyachenko, A. (2020b): Integration of reliability-based topology optimization into biomechanics: Application on hollow stems used in cementless total hip arthroplasty, VIII International Scientific and Practical Conference «Innovative technologies in science and education», ITSE 2020, Rostov-on-Don, Russia, 19-30 August 2020. E3S Web Conf., 210 (2020) 06003. https://doi.org/10.1051/e3sconf/202021006003

Ma, M.; Wang, L. (2021): Reliability-based topology optimization framework of two-dimensional phononic crystal band-gap structures based on interval series expansion and mapping conversion method, International Journal of Mechanical Sciences, 196, 2021, 106265.

Pérez-Rúa, J-A.; Lumbreras, S.; Ramos, A.; Cutululis, N.A. (2021): Reliability-based topology optimization for offshore wind farm col-lection system. Wind Energy, (2021), pp. 1–19. https://doi.org/10.1002/we.2660

De, S.; Maute, K.; Doostan, A. (2021): Reliability-based topology optimization using stochastic gradients, Structural and Multidisciplinary Optimization, 64(5), pp. 3089–3108 (2021).

Gao, Y.; Liu, Y. (2021): Reliability-based topology optimization with stochastic heterogeneous microstructure properties, Materials & Design, 205, July 2021.

Wang. L.; Li, Z.; Ni, B.W.; Gu, G. (2021a): Non-probabilistic Reliability-based Topology Optimization (NRBTO) Scheme for Continuum Structures Based on the parameterized Level-Set method and Interval Mathematics, Computer Methods in Applied Mechanics and Engineering, 373, 2021, 113477.

Wang, L.; Liu, Y.; Liu, D.; Wu, Z. (2021b): A novel dynamic reliability-based topology optimization (DRBTO) framework for continuum structures via interval-process collocation and the first-passage theories, Computer Methods in Applied Mechanics and Engineering, 386, 2021, 114107.

Wang, L.; Ni, B.; Wang, X.; Li, L. (2021c): Reliability-based topology optimization for heterogeneous composite structures under interval and convex mixed uncertainties, Applied Mathematical Modelling, 99, 2021, pp. 628-652.

Yin, F.; Dang, K.; Yang, W.; Ding, Y.; Xie, P. (2021): An efficient approach to reliability-based topology optimization for the structural lightweight design of planar continuum structures, Journal of Mechanics, 37, 2021, pp. 270–281. https://doi.org/10.1093/jom/ufaa019

Xia, H.; Qiu, Z.; Wang, L. (2021): Reliability-based topology optimization for freely vibrating continuum structures with unknown-but-bounded uncertainties, Structural and Multidisciplinary Optimization, 63, pp. 2751–2770 (2021).

Kharmanda, G.; Mulki, H.; Antypas, I. (2021): Linear versus nonlinear reliability-based topology optimization with application to bridge structures, XIV International Scientific and Practical Conference, Rostov-on-Don, Russia, February 2021, E3S Web of Conferences 273:04009. https://doi.org/10.1051/e3sconf/202127304009

Kharmanda, G.; Mulki, H.; Antypas, I. (2021): Nonlinearity investigation of reliability-based topology optimization strategies with application to total hip replacement, XIV International Scientific and Practical Conference, Rostov-on-Don, Russia, February 2021, E3S Web of Conferences 273(4):02004. https://doi.org/10.1051/e3sconf/202127302004

Madruga, A.R.M.; Prado, H.E.V.O; Simonetti, H.L.; Almeida, V.S.; Almeida, V.D.D. (2021); Reliability-based topology optimization using evolutionary methods for three-dimensional structures analysis, CILAMCE-PANACM-2021, Proceedings of the joint XLII Ibero-Latin-American Congress on Computational Methods in Engineering and III Pan-American Congress on Computational Mechanics, ABMEC-IACM, Rio de Janeiro, Brazil, November 9-12, 2021.

Pham, T.; Hoyle, C. (2021): Reliability-based topology optimization under random-field material, model. Preprint 2021. hal-03505301f. (https://hal.archives-ouvertes.fr/hal-03505301/document)

 