

Investigation of thermal and fluidic losses and their effect on the performances of a Stirling refrigerator

Influence des pertes thermiques et fluidiques sur les performances d'un réfrigérateur Stirling Résumé

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RÉSUMÉ. La conception d'une machine Stirling nécessite la compréhension des processus qui régissent leur fonctionnement pour prédire les paramètres de fonctionnement optimaux de la machine. Le modèle simple modifié utilisant les paramètres géométriques du modèle de moteur FEMTO-ST est appliqué dans cet article pour étudier l'influence des pertes en fonction de la fréquence de fonctionnement et de la pression de charge. Les variations des pertes de charge dans le régénérateur et les échangeurs de chaleur ont été étudiées en fonction de la fréquence de fonctionnement et de la pression de charge. La part des différentes pertes de puissances thermiques ainsi que leurs effets sur les performances de refroidissement ont été évalués aux différentes fréquences de fonctionnement et pressions de charge. Les pertes par frottement des fluides et les pertes dues aux imperfections du régénérateur augmentent avec la vitesse de rotation et la pression de charge.

ABSTRACT. The design of a Stirling machine requires the understanding of the processes that govern their operation to predict the optimum operating parameters of the machine. The modified simple model using the geometrical parameters of the FEMTO-ST engine model is applied in this paper to investigate the trend of losses with respect to operating frequency and charging pressure. The variation of pressure drops across the regenerator and heat exchangers have been investigated as functions of operating frequency and charging pressure in order to predict the extent of fluid friction losses. The share of different power and heat losses as well as their effects on cooling performance have been evaluated at the different operating frequencies and charging pressures. The share of fluid friction loss and loss due to regenerator imperfection increase with rotational speed and charging pressure.

MOTS-CLÉS. Réfrigérateur Stirling, Pertes de charge, Pertes de puissance, Performances, Régénérateur.

KEYWORDS. Stirling refrigerator, pressure drop, Power losses, Heat losses, Performance, Share of losses.

Nomenclature

A	cross sectional area, m^2	ϵ	regenerator effectiveness
A_{wgt}	wetted area of the metal, m^2	γ	ratio of specific heats, (C_p/C_v)
C	average molecular speed, $m.s^{-1}$	ω	angular speed, $rad.s^{-1}$
C_p	isobaric specific heat, $J.kg^{-1}.K^{-1}$	τ	compression ratio
C_v	isochoric specific heat, $J.kg^{-1}.K^{-1}$	θ	crank angle, rad
D	piston diameter, m	<i>Index and exponent</i>	
d	hydraulic diameter, m	c	compression space
f_r	Reynolds friction factor	cr	chiller
G	mass flux, $kg.m^{-2}.s^{-1}$	e	expansion space
J	annular gap between the cylinder and piston displacer, m	g	gas
K	heat conductivity, $W.m^{-1}.K^{-1}$	h	hot heat exchanger
L	length, m	r	regenerator
m	mass of working fluid, kg		
m_{leak}	mass leakage , kg		
n	rotational speed, rpm		
NTU	number of transfer unit		
N_{st}	Stanton number		
Q	heat, m		
P	pressure, Pa		
Pr	Prandtl number		
R	gas constant, $J.kg^{-1}.K^{-1}$		
Re	Reynolds number		
s	stroke , m		
T	temperature, K		
V	volume, m^3		
$W_{i,ad}$	ideal adiabatic work, m		
<i>Greek symbols</i>			
Δ	difference		

1. Introduction

The Stirling engine is one of a variety of external heat engines that can be used to generate power from various thermal energy sources. First, it was invented in 1816 by Robert Stirling. However, the benefits of Stirling engines have been revealed with the advancements in engineering and the demonstration of new methods to implement this thermodynamic cycle. It could be used in the reverse cycle as the Stirling refrigerator as it was first realized as a cooling machine in 1832 [1]. Due to these facts, Stirling machines are suitable alternatives for power generation and refrigeration in various applications. The Stirling cycle refrigerating machine designs are categorized as kinematic machines, where the piston and displacer are mechanically linked to the drive shaft, or free-piston machines, where the piston is coupled to the power supply by a linear motor and the displacer is driven by the gas pressure variation in the system. Appropriate design and modeling of Stirling machines will result in better performance as it will minimize irreversibilities associated with the operation and geometrical configuration.

The impact of losses has been investigated on the performance of irreversible solar-powered Gamma type Stirling engine [2]. They highlighted that the two main dissipations are regenerator imperfection loss and leakage loss in the engine. These losses are mostly dependent on geometrical and operational parameters. A combination of geometric and operating parameters was optimized for the Beta-type Stirling engine by Ahmed et al. [3]. In the research, it has been discussed that almost all losses increase with operating speed and charging pressure. However, the share of each loss have not been discussed.

Theoretical analysis and optimization of geometrical parameters for Stirling engines have been conducted [4]. In this research, it has been reported that alpha-type Stirling engines are less applicable for low-temperature difference, whereas Beta-type engine produced the highest shaft work.

A comprehensive review of the Stirling cooling machines has been conducted for a range of cooling applications [5]. The review have presented the general working principle, the configuration and drive mechanisms as well as research findings of Stirling refrigerators. Due to its wider application, high theoretical COP, as well as quieter and simpler operation, Stirling cycle cooler is worth researching. Different thermodynamic models for predicting the performance of the Stirling cooler were also proposed and developed in the past decades.

A Stirling refrigerator with a V-type configuration was approximated for a domestic cooling device with a charging pressure of 2 bar and compression ratio ranging between 1.5 and 2 [6]. The refrigerator performance parameters COP and cooling power showed a parabolic shape with a single optimal performance value with respect to operating speed [7]. Eid et al. [8] also investigate the effect of operating speed and they confirmed that cooling power has a single optimum value with respect to speed. It has been presented that, if engine speed increases too much, cycle efficiency will decrease as a result of the rapid increase in flow resistance and regenerator heat transfer losses [9].

The effects of rotational speed and charged pressure on different types of losses have been investigated for a Stirling engine [3, 9, 10]. All types of losses have been increased with an increase in operating speed as well as with charge pressure [3]. Heat conduction, shuttle heat, and flow resistance losses changed slightly as charged pressure increased [9]. Pressure drop loss, mechanical friction loss, and buffer space losses increased with rotational speed [10].

Due to losses associated with real-world conditions compared with ideal situations used for analytical

purposes, a practical Stirling refrigerator can't achieve a performance of the Carnot limit. A non-ideal second-order numerical model with losses has been developed for domestic Stirling refrigerator [11]. However, the effect and trend of each loss on the performance have not been investigated so far in detail for the Stirling cycle refrigerator. This research tries to analyze the share of different types of losses with respective categories and their effect on the overall performance of the Stirling cycle refrigerator.

The purpose of this paper is to analyze the percentage share of different losses and to investigate the effects of these losses on engine performance. Actually, this paper has been also presented to the national thermal conference [13].

2. Theory

The Stirling cycle machine consists of two variable volumes (compression and expansion) spaces physically separated by a regenerator and three constant volumes. For the Stirling cycle refrigerator, heat is absorbed from the low-temperature heat source, part of the heat is stored and released in the regenerator, and rejected to a hot heat sink (see Fig 1).

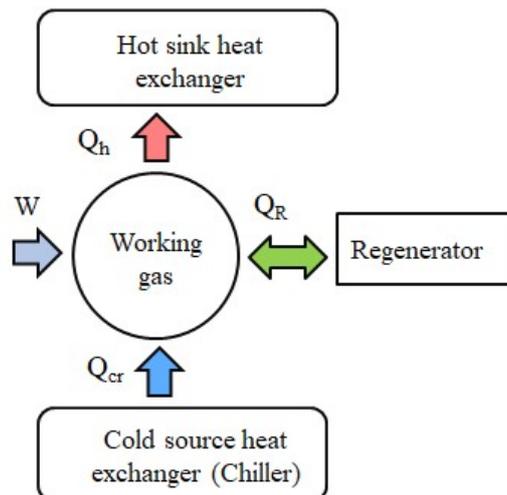


Figure 1.: Schematic diagram of Stirling refrigerator.

The four thermodynamic transformations for an ideal reversed Stirling cycle includes :

1. An isothermal compression where heat is rejected to the heat sink (hot heat exchanger).
2. Isochoric cooling where the gas transmits heat to the regenerator.
3. An isothermal expansion where heat is absorbed from the cold source (chiller).
4. Isochoric heating where the heat stored in the regenerator is transmitted to the working fluid.

A non-ideal numerical model has been developed in the author's previous article [11]. In such a thermodynamic model, the overall Stirling refrigeration machine is configured into five control volumes (two working spaces and three heat exchangers) serially connected similar to works of Urieli and Berchowitz [12]. The overall approach for driving the equation set that could represent for the analysis of such type of machine is to apply energy conservation equation and equation of state for each control volume and link with the equation of conservation of mass across the whole refrigerating machine. In

this paper, such a model is used to evaluate the share of different losses and their effect on the cooling performance of the Stirling refrigerator.

2.1. Numerical model

The performance of Stirling machine is strongly dependent on geometrical and physical parameters such as dimensions, heat transfer coefficients, heat source temperatures and regenerator characteristics. Predicting the performance of Stirling machine is important for the design of the machine. The ideal Stirling cycle has been already studied by several researchers; however the performance of real Stirling machine is far from the ideal Stirling cycle.

The system of ideal adiabatic differential equations was modified as described by author's previous work [11] through including the effects of gas leakage from working space to buffer space (crankcase) and shuttle heat loss by displacer from compression to expansion spaces for the Stirling cycle refrigerating machine. The summary of modified ideal adiabatic equation is presented as shown in Table 1. The derivative operator D based on [12] with respect to time where ($DX = \frac{dX}{dt}$) and X is a thermodynamic parameter (P, V, T, W, Q, m).

Parameters	equations
Pressure	$DP = \frac{-P\gamma(\frac{DV_c}{T_{ch}} + \frac{DV_e}{T_{cre}}) + \gamma R \frac{DQ_{shut}}{C_p} (\frac{T_{ch}-T_{cre}}{T_{ch}T_{cre}}) - 2\gamma R Dm_{leak}}{\frac{V_c}{T_{ch}} + \gamma(\frac{V_h}{T_h} + \frac{V_r}{T_r} + \frac{V_{cr}}{T_{cr}}) + \frac{V_e}{T_{cre}}}$
Change of mass	$Dm_i = m_i \frac{DP}{P} = \frac{DP}{R} \frac{V_i}{T_i} \text{ (where } i = h, r, cr \text{)}$ $Dm_c = \frac{PDV_c + \frac{V_c DP}{\gamma}}{RT_{ch}} + \frac{DQ_{shut}}{C_p T_{ch}}$ $Dm_e = \frac{PDV_e + \frac{V_e DP}{\gamma}}{RT_{cre}} - \frac{DQ_{shut}}{C_p T_{cre}}$ $DQ_{shut} = \frac{\pi s^2 K_g D_d}{8 J L_d} (T_c - T_e)$
Mass flow	$\dot{m}_{ch} = -Dm_c - \frac{DQ_{shut}}{C_p T_{ch}} - Dm_{leak}$ $\dot{m}_{hr} = \dot{m}_{ch} - Dm_h, \quad \dot{m}_{rcr} = \dot{m}_{cre} + Dm_{cr}$ $\dot{m}_{cre} = Dm_e - \frac{DQ_{shut}}{C_p T_{cre}}$ <p>Where, $Dm_{leak} = d_p \pi \frac{P + P_{buffer}}{4RT_g} (U_p J - \frac{J^3}{6\mu} (\frac{P - P_{buffer}}{L_p}))$</p>
Temperature variation	$DT_e = T_e (\frac{DP}{P} + \frac{DV_e}{V_e} - \frac{Dm_e}{m_e})$ $DT_c = T_c (\frac{DP}{P} + \frac{DV_c}{V_c} - \frac{Dm_c}{m_c})$
Energy and power	$DQ_{h,i} = \frac{V_h DPC_v}{R} - C_p (T_{ch} \dot{m}_{ch} - T_h \dot{m}_{hr})$ $DQ_{r,i} = \frac{V_r DPC_v}{R} - C_p (T_h \dot{m}_{hr} - T_{cr} \dot{m}_{rcr})$ $DQ_{cr,i} = \frac{V_{cr} DPC_v}{R} - C_p (T_{cr} \dot{m}_{rcr} - T_{cre} \dot{m}_{cre})$ $DW_{i,ad} = DW_c + DW_e \text{ where}$ $DW_c = PdV_c \text{ and } DW_e = PDV_e$

Table 2.1.: Summary of modified ideal adiabatic model (adapted from [11]).

Then, the model was further corrected to modified simple model by including power and heat losses. These losses include regenerator imperfection loss, fluid friction loss, mechanical friction, conduction heat loss in the regenerator wall, pressure drop due to finite speed of piston, conduction heat loss, and

gas spring hysteresis loss as shown in Table 2. The details of these losses are also presented in the authors' previous work [11].

Losses	equations
Loss due to regenerator imperfection	$Q_{rl} = mc_p(1 - \epsilon)(T_c - T_e)$ $\epsilon = \frac{NTU}{NTU+1}, NTU = N_{st} \frac{A_{wg}}{A}$ $N_{st} = 0.023Pr^{-0.6}Re^{-0.2}$
Losses due to pressure drop in heat exchangers	$W_{fr} = \int_0^{2\pi} (\Delta P \frac{dV_e}{d\theta}) d\theta$ <p>where $\Delta P = \Delta P_h + \Delta P_r + \Delta P_{cr}$ and</p> $\Delta P_i = \frac{2f_r \mu V_i G_i l_i}{m_i d_i^2} \text{ (Where } i = h, r, cr)$
Mechanical friction losses	$W_{mec.fr} = 2\Delta P_{mec.fr} V_{swc}$ $\Delta P_{mec.fr} = \frac{(0.94+0.045sn)10^5}{3(1-1/3\tau)} (1 - 1/\tau)$
Conduction heat loss in the regenerator wall	$Q_{wrl} = k \frac{A_{wg}}{L_f} (T_{wh} - T_{wcr})$
Heat conduction losses	$\dot{Q}_{cond} = k \frac{A}{L} \Delta T$
Losses due to finite speed of piston	$W_{fin-sp} = 2\Delta p_{fin.sp} V_{swc}$ <p>and, $\Delta p_{fin.sp} = \frac{1}{2} (P \frac{aU_p}{C_c} + P \frac{aU_p}{C_e})$</p> $a = \sqrt{3\gamma} \text{ and } C = \sqrt{3RT}$
Gas spring hysteresis losses	$\dot{W}_{hys} = \sqrt{\frac{1}{32} \omega \gamma^3 (\gamma - 1) T_w P_{mean} k_g (\frac{V_d}{2V_t})^2} A_{wg}$

Table 2.2.: Summary of additional losses included.

The numerical model presented above, was evaluated by considering the FEMTO 60 Stirling engine in which the main parameters and dimensions of the device are tabulated in Table 3 operating as a refrigerating machine as a case study.

No	Parameters	value
1	Hot heat temperature (K)	305
2	Cooling temperature (K)	270
3	Piston diameter (mm)	60
4	Piston stroke (mm)	40
5	Regenerator length (mm)	50
6	Diameter of regenerator (mm)	82
7	Compression space swept volume (cm ³)	103
8	Expansion space swept volume (cm ³)	113
9	Working gas	Nitrogen
10	Frequency (Hz)	7.5
11	Charging pressure (bar)	17.5

Table 2.3.: Main specifications of Stirling cooling machine used in this study (details can be found [14]).

3. Results and discussion

In this section, the shares of different types of losses associated with the Stirling cycle with the respective category are presented. The variation of pressure drops across the regenerator and heat exchangers have been investigated as functions of operating frequency and charging pressure in order to predict the extent of fluid friction losses. Furthermore, the trends of these losses with operating speed are discussed. These results are based on simulation findings of the second-order numerical model described so far using air as a working fluid.

Fig 2a and b demonstrates the effect of operating frequency on pressure drop in the regenerator and two heat exchangers respectively for operating frequency of 6 Hz and 12 Hz. It can be confirmed that the pressure drop increases with frequency. This is due to the fact that as frequency increases friction between flowing working fluid and heat exchanger solid surface increases as also described by [15]. For instance, the maximum pressure drop in regenerator increases from 0.81 bar to 2.83 bar which is 252% increase as frequency increases from 6 Hz to 12 Hz. Furthermore, the pressure drops in chiller and hot heat exchanger (HHX) increase by 230% and 233% respectively as operating frequency increases from 6 Hz to 12 Hz. Generally, operating frequency has critical impact on the overall pressure drop across heat exchangers. Most importantly, the pressure drop especially in regenerator has a large share on the total pressure drop and it also increases at higher rate with frequency.

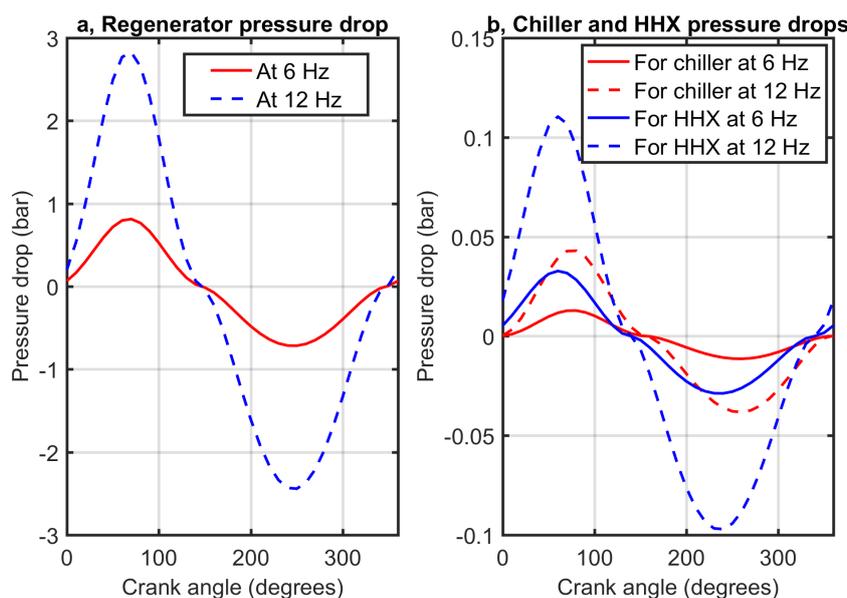


Figure 2.: Effect of machine operating frequency on the pressure drop in heat exchangers.

The effect of charging pressure on the pressure drop across regenerator and heat exchangers have been also investigated as shown in Fig 3, where the pressure drops of regenerator as well as other heat exchanges of a Stirling refrigerator are compared for the cases of 15 bar and 25 bar. From Fig 3a, it could be observed that the pressure drop in the regenerator increases by 53% as pressure increase from 15 bar to 25 bar. Similarly, Fig 3b demonstrates the pressure drops in chiller and hot heat exchanger (HHX) increases by 46.5% and 47.3% respectively as charging pressure increase from 15 bar to 25 bar. Hence, the pressure drop in regenerator has a large share on overall pressure drop and it also increases at higher rate with pressure. It means the fluid friction power loss increases with increase in charging especially due to pressure drop in regenerator.

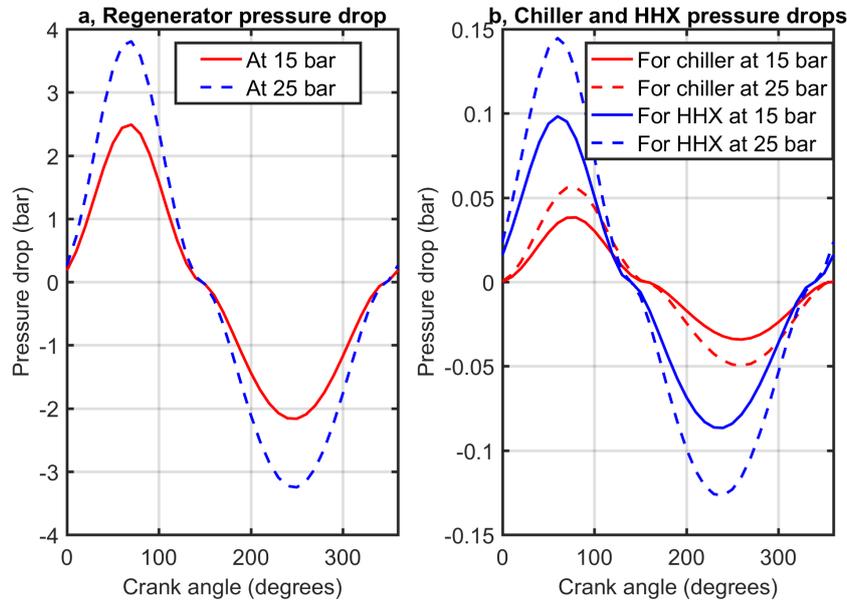


Figure 3.: Effect of charging pressure on the pressure drop in heat exchangers.

Fig 4a and b demonstrate the percentage of different power and heat losses with respect to total losses at respective categories at a charging pressure of 17.5 bar. In Fig 4a, the average share of power loss is presented. It is clear that from the considered losses, the major share is mechanical friction loss (48%), followed by fluid friction loss due to pressure drop (40%). Others cover only 12% of the total power loss. Similarly in Fig 4b, the average share of heat loss is presented. The major heat loss is due to regenerator imperfection which comprises 74%. The second major heat loss is conduction heat loss (14%) across the displacer due to the temperature difference between the two working spaces. This type of loss is basically more dependent on the temperature difference and hence the share may vary.

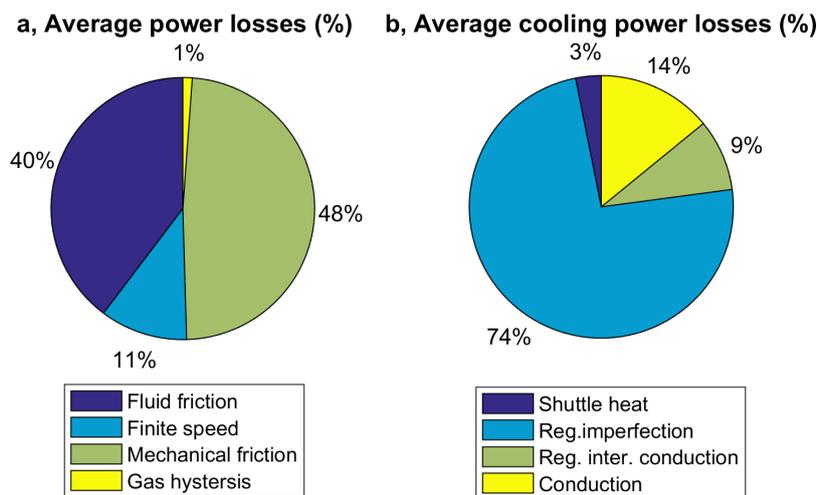


Figure 4.: Analysis of the percentage of losses at a pressure of 17.5 bar.

Fig 5a and b show the percentage of different power and heat losses with respect to total losses at respective categories and at a charging pressure of 25 bar. Fig 5a, demonstrates the percentage share of different power losses and Fig 5b, show the percentage share of various heat losses. According to Fig

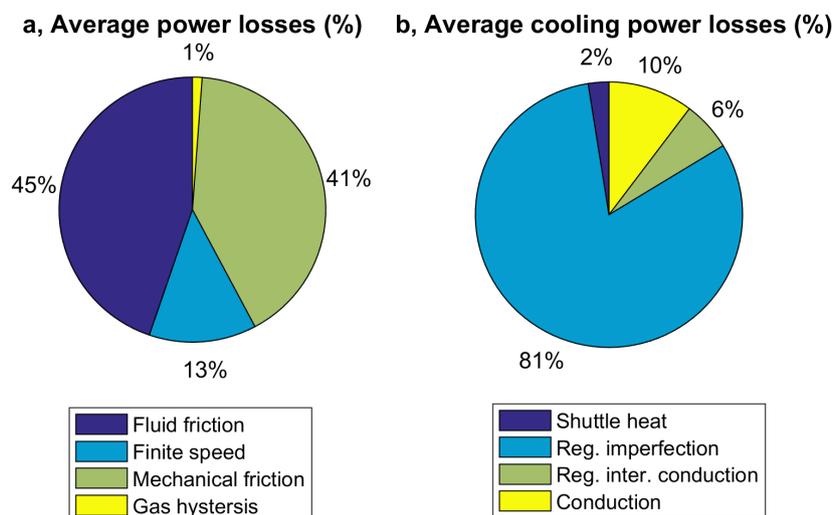


Figure 5.: Analysis of the percentage of losses at a pressure of 25 bar.

4, it is noticed that the percentage share of fluid friction loss and loss due to regenerator imperfection increase with increase charging pressure. Great care should be given to minimization of fluid friction loss and regenerator imperfection for machines operating at higher pressure.

Fig 6a,b,c, and d illustrate the effect of rotational speed on the major power losses, major heat power losses, cooling power, and COP respectively at an ambient temperature of 300 K, chiller temperature of 270 K, and charging pressure of 17.5 bar. As demonstrated in Fig 6a, except for gas hysteresis power loss all power losses are mainly affected by rotational speed. Fluid friction loss increases strongly at a higher speed. As it is shown in Fig 6b regenerator imperfection loss linearly increases and all other heat power losses relatively remain unchanged with the operating frequency. From Fig 6c, it could be seen that cooling power loss increases with an increase in rotational speed. This is mainly due to the increase in the thermodynamic cycle per unit time. Lastly, as it is seen in Fig 6d, COP has an optimum value with an increase in rotational speed. This is expected as demonstrated in Fig 6a and b, both power and heat power losses increase which resulted in lower COP at a higher speed.

4. Conclusion

The design and development of Stirling cycle refrigerators require an understanding of the processes that govern the operation of the machine. The operating condition and processes of the Stirling cycle refrigerator are accompanied by different types of losses. A non-ideal thermodynamic model is used to investigate the share of different losses and the trend of such losses with the operating speed and charging pressure. The variation of pressure drops across the regenerator and heat exchangers have been investigated as functions of operating frequency and charging pressure in order to predict the extent of fluid friction losses. The major power and heat losses, as well as their effect on the performance of the cooling machine, have been discussed. Fluid friction power losses and regenerator imperfection losses are found as the two major losses that are mostly affected by charging pressure and operating speed. The shares of these two losses over their respective total losses increase with an increase in operating

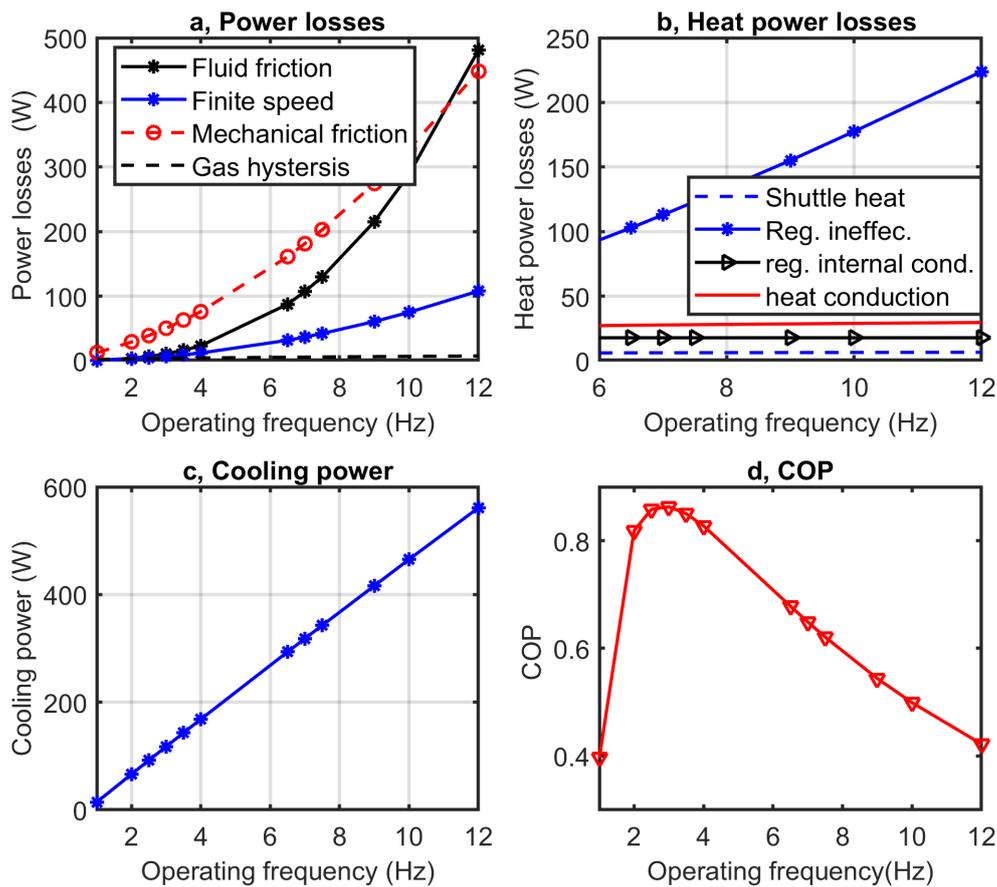


Figure 6.: Trend of major losses and refrigeration performance with respect to the operating frequency.

speed as well as with charging pressure. The coefficient of performance of the refrigerating machine decreases at higher operating frequency primarily due to the increase of fluid friction and regenerator imperfection losses. Hence, during the design of the Stirling refrigerator, critical investigation of losses shall be conducted for the intended operating range of the machine to minimize major losses and the summation of overall losses. Then, better performance could be achieved.

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