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The Vitruvian Man, and the Geometry of Life According to Leonardo da Vinci

L'Homme de Vitruve et la géométrie du vivant de Léonard de Vinci

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ABSTRACT. This study aims to understand the construction by Leonardo da Vinci of the Vitruvian Man, which represents the graphic resolution of an ancient challenge set by the Roman architect Marcus Vitruvius Pollio in his treatise De Architectura. It seeks to explain the genesis of this work, its progressive elaboration, and the way in which Leonardo succeeded in addressing a problem that had remained unanswered for centuries. The analysis also highlights the seminal role of mathematics, which since the earliest civilizations has been regarded as a universal language of knowledge and perfection. Long considered to disclose the secrets of the universe, its use in this emblematic drawing reveals Leonardo's scientific spirit, driven by the pursuit of universal harmony.

RÉSUMÉ. Cette étude est consacrée à l'étude du dessin de *l'Homme de Vitruve* de Léonard de Vinci, ce qui nous permet de dévoiler la géométrie sous-jacente à la construction dudit homme, défi antique lancé par l'architecte romain, Marcus Vitruvius Pollio, dans son ouvrage *De architectura*. Notre étude vise à comprendre la genèse de cette œuvre, son élaboration progressive et la manière dont Léonard a su élucider un problème resté sans réponse durant des siècles. L'analyse met également en lumière le rôle fondateur des mathématiques érigées depuis les premières civilisations en langage commun de la connaissance et de la perfection. Considérées, depuis toujours, comme un moyen de percer les secrets de l'univers, leur usage dans ce dessin emblématique, révèle l'esprit scientifique de Léonard animé par la recherche d'une harmonie universelle.

MOTS-CLÉS. Léonard de Vinci, L'homme de Vitruve, Mathématiques, Harmonie, géométrie ancienne.
KEYWORDS. Leonardo da Vinci, Vitruvian Man, Mathematics, Harmony, Ancient geometry.

1.The continuum of knowledge from the ancient Orient to the Renaissance

1.1. *The cradles of wisdom*

Since the beginning of time, human beings have questioned the origin of the world and the order that governs it. The earliest accounts were mythical. They staged gods who embodied the forces of nature. But alongside these sacred narratives, little by little, forms of knowledge emerged that were based on observation, calculation and experience. This new quest guided by reason inaugurated a new way of understanding the world and it would cross the entire history of humanity, from Mesopotamia to Egypt, from Persia to India and China, then from Greece to Rome and from the Orient to the medieval West.

Thus, was woven, across time, a true chain of knowledge, in which each civilization assimilated, transformed and enriched the heritage of the previous one. Over the centuries, the ancient Orient, Greece and Rome transmitted to the Renaissance a body of knowledge that would find in Leonardo da Vinci an exceptional heir. This is why, in order to understand the drawing of the Vitruvian Man and Leonardo's scientific work, it is first necessary to retrace this intellectual genealogy. It's therefore fitting to return to the most ancient sources, in Mesopotamia and Egypt, where the first knowledge about the sky, numbers and proportions was born.

One of the very first scholarly civilizations was Mesopotamia, cradle of the Sumerians and Babylonians. As early as the second millennium before our era, priest-astronomers observed the sky night after night. For them, scrutinizing the stars meant interpreting the will of the gods, Venus was

Ishtar, Jupiter was Marduk, the Moon was Sîn[1]. Their observations were recorded on clay tablets, as evidenced by the archives found at Uruk and Babylon[2]. These records not only made it possible to predict certain celestial phenomena, such as eclipses, but also to regulate agricultural life which depended directly on celestial cycles. Astronomy was thus inseparable from the organization of the city.

These tablets gave rise to the Babylonian ephemerides[3], genuine journals of the sky that recorded observed phenomena day after day. These journals served as the basis for calculations and predictions. Such systematic tracking was made through the adoption of the sexagesimal system[4]. Unlike our decimal system based on groups of ten (units, tens, hundreds), the Mesopotamians reasoned in groups of sixty. This choice simplified calculations of fractions and proportions. Its legacy survives today in the division of time (60 minutes, 60 seconds) and of space (360 degrees for the circle).

This numerical system allowed scribes to develop an advanced mathematical culture. They established numerical tables, were able to calculate square roots and solve simple equations. This knowledge constituted a true science of numbers. Thus, Mesopotamia appears as the matrix of an intellectual approach that would mark all of history. Behind the gods and myths, it was indeed through numbers that humanity began to understand the order of creation.

In the Middle East, between 2000 and 1700 BC, and then during the New Kingdom from 1550 to 1070 BC, the same desire to decipher the laws of nature appeared in the Nile Valley. Heir in part to Mesopotamian traditions coming through the Near East, but also to older African cultures rooted in the river basin and sub-Saharan regions, Egypt developed its own observations and experiments.

Within this vast interlacing was born a way of thinking in which the movement of the stars and the rhythm of the river formed a language, guarantor of the order of the world[5]. As in Mesopotamia, the great forces of the universe were embodied by gods. The regular floods of the Nile, synonymous with fertility, were perceived as a blessing. Conversely, a flood that was too low or too high, a drought or a famine could be interpreted as a divine punishment[6]. But little by little, the Egyptians understood that the climatic and agricultural disturbances that they interpreted as the gestures of the gods were only regular phenomena that had to be observed closely to maintain the balance of life. This means that, it was by observing the star Sothis (Sirius) that the Egyptians discovered that its heliacal rising, that is, the moment when it became visible again at dawn after a period of invisibility, coincided with the beginning of the flooding of the Nile[7]. This astronomical marker signaled the beginning of the year and enabled the Egyptians to develop the first solar calendar of 365 days[8].

The priest-mathematicians of Egypt had also developed an advanced geometry and arithmetic, as evidenced by the Rhind papyrus (around 1650 BC), written by the scribe Ahmès[9]. Thus, like Mesopotamia, the Egyptians transformed a religious interpretation into mathematical and astronomical knowledge, with the conviction that the universe, to be understood and mastered, had to be measured.

1.2. *Greek Antiquity and rational thought*

For millennia, the Egyptians, the Mesopotamians and the Persians had accumulated knowledge about the sky, numbers and proportions, to this must be added the Indian and Chinese traditions. In India, the Śulbasūtras (around 800–500 BC) already contain geometric statements close to the Pythagorean theorem, while in China, the Zhou Bi Suan Jing (3rd century BC) bears witness to advanced astronomical and geometric calculations[10]. This knowledge, which circulated through trade routes, travel and conquests, constituted a shared heritage.

It was from this universal foundation that ancient Greece was nourished, which transformed part of these practices into a true rational science. Heir to oriental observation, it no longer contented itself with predicting and measuring celestial phenomena but sought to understand their causes and to demonstrate them through reasoning.

Therefore, begins with Thales of Miletus (around 624–546 BC) the first great stage of rational thought in the West. Considered by Aristotle (384–322 BC) as one of the first physicists[11], Thales embodies the transition between oriental knowledge and Greek philosophy. According to the Greek historian Herodotus[12], he is said to have travelled to Egypt, where he received instruction from the priests in the art of measuring the earth and observing the stars. This stay is also mentioned by the historian and biographer Diogenes Laërtius[13], who reports that Thales returned from his journey with new methods of calculation and observation. Tradition holds that he measured the height of the pyramids by using their shadow. He waited until his own shadow equaled his height and deduced by proportion that of the pyramid. This procedure, based on the similarity of triangles, became one of the major principles of geometry. Thales thus appears as the first to have shown that the intangible could be measured through mathematical reasoning.

Several centuries later, Leonardo da Vinci (1452–1519) applied this same logic in his studies on light (figure 1). He drew a large arc that represents an illuminated surface, and three cones placed below that depict an object facing forward and two objects positioned to the sides. The rays coming from the edges of the arc delimit the illuminated area. It is the similar triangles that represent the lateral objects which make it possible to compare the quantity of light received according to position. Thus, Leonardo measured light with the same geometric logic as Thales.

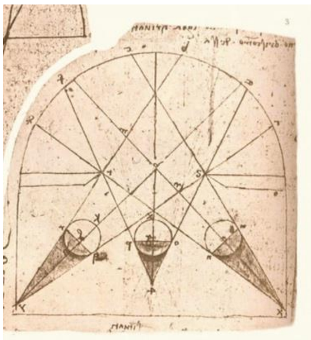
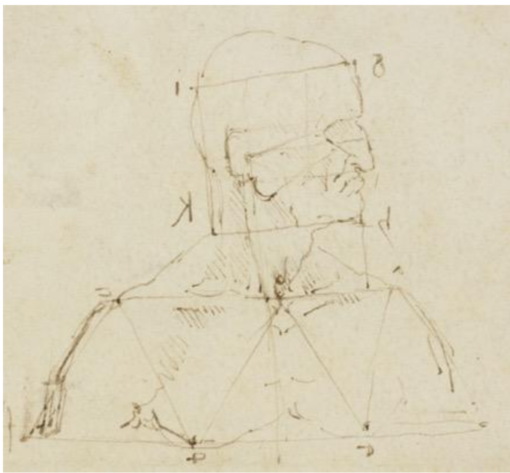


Figure 1. *Studies on the theory of light and shadows*
Manuscript A, Institut de France
Da Vinci

For Leonardo, geometry constituted the principal tool for the understanding of phenomena and guided the conduct of all his research. In his anatomical studies, he decomposed living forms into networks of triangles. This geometric grid applied in his quest for the ideal man and horse enabled him to compare and understand the proportional relationships that govern their measurements (figures 2). In both cases, the method employed is based on a principle that demonstrates the influence of Thales and Leonardo's desire to unify the study of living beings within a universal geometric language.

Study of horse proportions
RCIN 912318, Codex Windsor



Study of human proportions
RCIN 912607, Codex Windsor

Figures 2. *Da Vinci*

Among the heirs of the oriental and Greek tradition, Pythagoras of Samos (around 570–around 495 BC), philosopher, mathematician and founder of a spiritual community, was one of the most influential figures of Antiquity. The philosophers Iamblichus[14] and Porphyry[15] (3rd and 4th centuries) reported that he travelled to Egypt and Babylonia, where he came into contact with the Egyptian priests and the Chaldeans, heirs to long astronomical and mathematical traditions. He was initiated in Egypt into priestly practices and geometric knowledge, and is also said to have received the teaching of the Persian magi.

At the heart of his thought lies the idea that numbers and numerical ratios express the deep structure of the world. For Pythagoras, every phenomenon can be described through proportional relationships. This conception finds an illustration in the Pythagorean theorem, according to which, in a right triangle, the sum of the squares of the two sides adjacent to the right angle is equal to the square of the hypotenuse. The same logic is manifested in the discovery of musical proportions, the octave (2:1), the fifth (3:2) and the fourth (4:3), which show that the intervals most harmonious to the ear also obey simple ratios of the lengths of vibrating strings. Thus, for the Pythagoreans, mathematics represented a divine language, capable of revealing the hidden order of the cosmos[16].

This heritage reached Leonardo, who explicitly took up the analogy between music and geometry[17]. On several pages of his notebooks are found studies of organ pipes, vibrating strings and mechanical systems designed to produce regular sequences (figure 3). He compared the vibration of strings and the production of sound to numerical ratios. He observed that the pitch of the sound varies according to the length and tension of the string, thus confirming experimentally the principles of proportionality already stated by Pythagoras.

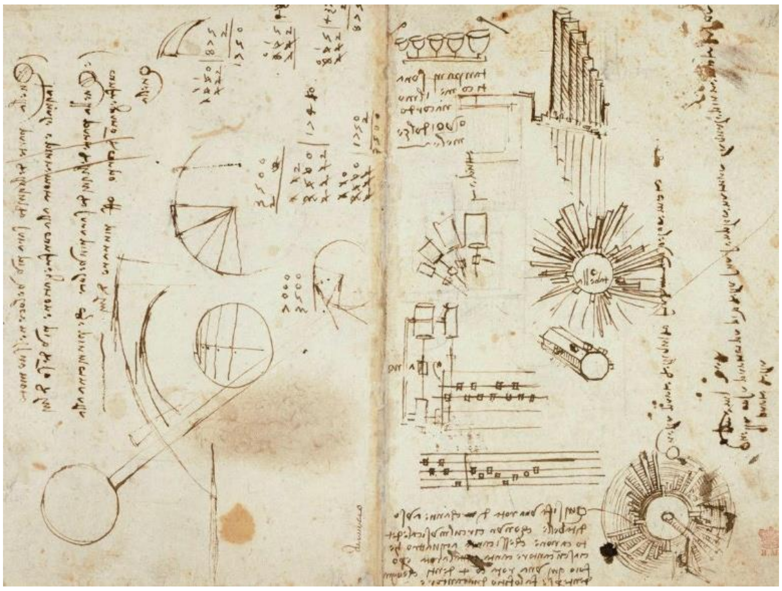
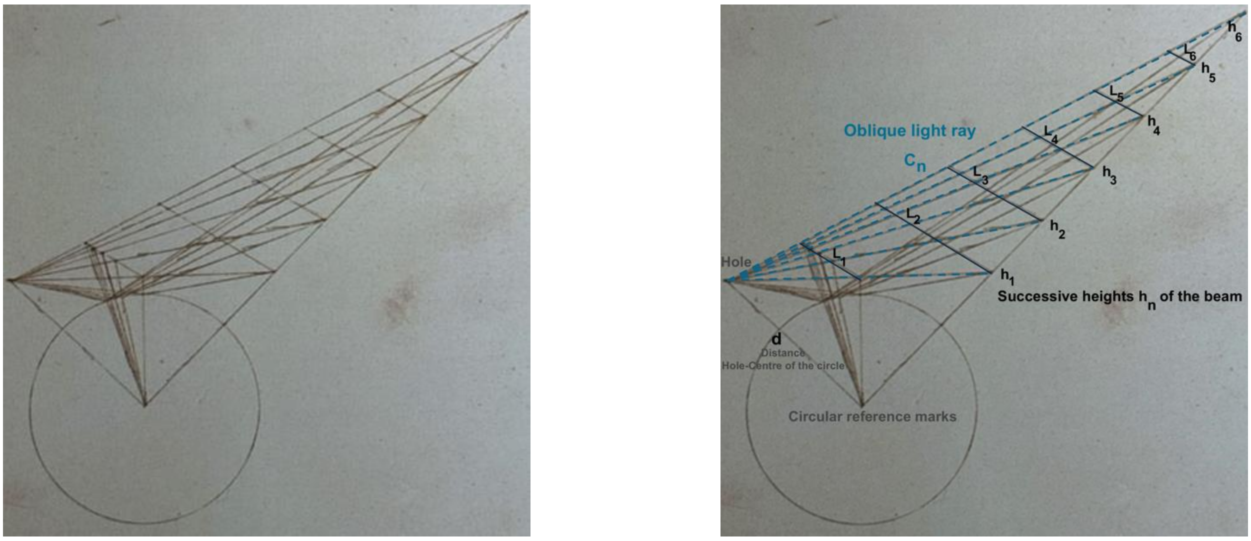


Figure 3. *Music studies, Codex Arundel, f.133v, f.133r, Da Vinci*

Similarly, Leonardo used the right triangle as a universal tool, capable of reducing all natural phenomena to measurable ratios. This approach is particularly evident in his research on optics.

In a study on the propagation of light, Leonardo drew a beam of light in the form of a cone (figures 4). He represented rays coming from a celestial body which, passing through a small opening, form behind this hole a beam of light. From this point, these rays unfold by dividing the beam into six successive right triangles. This division into six sectors is part of the tradition inherited from the sexagesimal system, used in medieval astronomy and optics[4]. And as the construction is based on right triangles, Leonardo naturally set the opening angle of the beam at 90°, that is, a quarter of the circle.



Figures 4. *Study on optics, the propagation of light, Codex Arundel I, Da Vinci*

To describe this construction geometrically, let d denote the distance between the opening and the center of the circular surface which serves as a spatial reference to allow the visualization and measurement of the opening of the luminous cone. Let h_n denote the height of the level and c_n the oblique length of the light ray corresponding to this level. The distance d is equal to the first height h_0.

Each level of the beam thus has its own oblique ray, which constitutes the hypotenuse of the right triangle formed with d and h_n. The configuration then obeys the Pythagorean theorem.

$$h_n^2 + d^2 = c_n^2$$

From one level to the next, the height h_n increases by a constant value, while the oblique length c_n grows more and more slowly. The larger the size of the right triangles becomes, the more the distance between the light rays narrows. This decrease of the ratios c_{n+1}/c_n is due to the function $c_n = \sqrt{(d^2+h_n^2)}$, which is increasing and concave. Thus, when h_n increases by a constant amount, the increments $c_{n+1} - c_n$ gradually become smaller, which is characteristic of a concave curvature.

This internal geometry corresponds to a physical phenomenon that Leonardo stated in several manuscripts, in the Codex Atlanticus, f.337 r. he noted:

« La luce si indebolisce secondo la distanza. »

« Light decreases as a function of distance »

On folio 126r. of this same manuscript :

«Ogni cosa veduta da lunga distanza perde di vigore e di chiarezza. »

« Anything seen from a distance loses strenght and clarity »

And in manuscript A, f. 17 v., of Institut de France :

«La luce si diminusce quanto più si allontana dalla sua origine. »

« Light diminushes as it moves away from its origin »

Thus, the rays c_n, progressively longer, carry a gradually weakened light, which visually reinforces the narrowing effect observed in the beam.

To make this natural mechanism visible, Leonardo constructed a true geometric grid within the beam of light. He began by dividing the first right triangle in two. He drew a segment from the center of the circle to the midpoint of the hypotenuse, perpendicular to h_n. From the midpoint of the hypotenuse, he drew a segment L_1 up to the larger segment c_n, the hypotenuse of the cone, in such a way that the rays of light were divided into two equal parts.

At each level h_n, an analogous segment L_n is drawn to the largest light ray c_n. These transversals then divide each ray into an increasing number of equal parts, two for the first, three for the second, four for the third and so on up to 7.

This regular subdivision reveals a concave progression on each light ray. The successive intersection points correspond to the fractions: 1/2, 2/3, 3/4, 4/5, 5/6, 6/7. Each of these values is greater than the preceding one, but according to increments that decrease regularly. This is the signature of a concave growth, perfectly understood by Leonardo.

To complete his construction, Leonardo projected from the center of the circle segments (see green arrow in figure 4) to the segment parallel to d at the point where the hypotenuses of the right triangles intersect. The intersection points of these segments on the circle were then connected by segments to the opening (see red arrow in figure 4). Finally, from these same intersection points, he drew segments to the summit h_n of each right triangle (black arrow in figure 4). These projections make it possible to visualize how the angles narrow and how the light weakens within the beam.

Thus, through the application of the Pythagorean theorem, Leonardo sought to understand the behavior of light by observing the successive variations of the rays. This study based on the concavity of the function $\sqrt{(d^2 + h_n^2)}$, constitutes a remarkable anticipation of differential calculus, long before its formalization by Newton and Leibniz in the 17th century[18].

At the same period, while certain philosophers explored numbers and figures to decipher the mysteries of creation, the field of medicine traced a new path of knowledge oriented toward the understanding of the human body. Hippocrates (around 460–370 BC), known as the “father of medicine”, broke with religious explanations of illness and affirmed that the body obeys the same laws as nature[19]. Health was not a gift from the gods, but the result of a fragile balance.

This idea was rooted in an older heritage. In Egypt already and in Persia, life was perceived as a play of balance between the four elements of nature. Empedocles of Acragas (around 490–430 BC) gave a first philosophical formulation by associating air, water, fire and earth with the birth and transformation of beings[20]. Hippocrates expressed this conception of natural balance in his famous theory of the four humors, according to which the human body is composed of four fundamental substances: blood, yellow bile, black bile and phlegm. Each of them corresponds to an element of nature, air, fire, earth and water, and is defined by an essential quality, hot, cold, dry or moist. Blood, associated with air and characterized by heat and moisture, symbolizes vitality and joy. In harmonious proportion it produces a lively and generous temperament, but in excess it causes agitation and instability. Yellow bile, linked to fire, hot and dry, represents strength of action and courage. Balanced it nourishes determination, but too abundant it generates anger and irritability, hence the french expression *“se faire de la bile”*, which conveys the idea of anxiety and agitation. Black bile, attached to earth, cold and dry, evokes depth of mind and sensitivity. When it dominates it plunges the individual into melancholy and sadness, a word moreover derived from the Greek *“melaina chole”* meaning *“black bile”*. As for phlegm, associated with water, cold and moist, it embodies stability and inner calm. Properly proportioned it makes it possible to keep one’s composure, but when it accumulates it causes slowness and inertia, which gave rise to the french expression *“garder son flegme”*, which denotes emotional restraint and self-control. Thus, in Hippocratic thought, the health of the body depended on the harmonious balance of these four humors, reflection of the very harmony of nature[21]. Illness appeared when one of them became excessive or insufficient. Thus, just as the world lives from the balance of the elements, man lives from the balance of his humors. This doctrine, founded on observation and experience, made medicine an empirical science attentive to symptoms and to the evolution of diseases[22].

This vision was extended by Galen of Pergamon (129–around 201 AD), a Greek physician in the service of the Roman Empire. An admirer of Hippocrates, he systematized the theory of the humors into a true functional anatomy based on animal dissections. His authority remained uncontested for more than a millennium, the schools of medieval and Renaissance medicine taught above all Galen, whose treatises were translated into Latin and Arabic[23]. But in the absence of systematic human autopsy, he retained major anatomical errors. It was precisely this fixed knowledge that Leonardo, centuries later, would challenge through direct experience.

Leonardo began to take an interest in anatomy as early as the 1480s in Florence, where he drew from living models. However, his true campaign of dissection did not begin until between 1506 and 1513. During the winter of 1507–1508, at the hospital of Santa Maria Nuova in Florence, he performed the autopsy of a centenarian who had died "without pain"[24]. In his sheets, today preserved in Windsor, he recorded unprecedented pathological observations on the vessels of the brain and the heart.

He shared with thinkers of Antiquity, notably Hippocrates, the conviction that man is a microcosm. In his thought, which he expressed in the Codex Atlanticus, f.80r., the human being and the world are but two expressions of the same organization, governed by universal laws.

«L'omo è detto da' antichi mondo minore; e certo questa denominazione è ben data, perché, come il corpo della terra è composto d'acqua, d'aria, di terra e di fuoco, così questo corpo dell'omo è fatto di queste medesime cose. E come l'omo ha in sé ossa che sostengono la carne, e la terra ha le rupi che sostengono la terra, come nell'omo le vene e i fiumi, così nel mondo; come nell'omo il sangue, così nel corpo della terra l'acqua, e come nell'omo il polso e il batter del cuore, così la terra ha il flusso e reflusso del mare.»

«*The man is called by the ancients the lesser world; and certainly this denomination is well given, because, as the body of* the *earth is composed of water, air, earth, and fire, so this body of the man is made of the same elements. And as man has bones in him that support the flesh and the earth has rocks that support the earth, as in man the veins and rivers, so in the world; as in man blood, so in the body of the earth water, and as in man the pulse and the beating of the heart, so the earth has the ebb and flow of the sea.* »

His late anatomical notes, preserved in the Codex Atlanticus, Leonardo further broadened this conception, inspired by Ptolemy's cosmographic vision, he conceived the human body as a small world (minor mondo), organized according to the same principles as the universe, he mentioned it on folio 327 r:

«Adunque qui con quindici figure intere ti sarà mostrata la cosmografia del minor mondo col medesimo ordine che innanzi a me fu fatto da Tolomeo nella sua cosmografia, e così dividerò poi quelli in membra, come lui divise il tutto in provincie.»

«Thus, with fifteen complete figures, the cosmography of the small world will be shown to you according to the same order that Ptolemy adopted before me in his Cosmographia; and I shall then divide the body into members, as he divided the whole into provinces»

Leonardo also adopted the thought of Hippocrates and Galen according to which the health of the body depends on the harmony of the four humors. He thus placed himself in the continuity of the ancient tradition while renewing it through direct observation and experimentation. His research belonged to a unified conception of living beings, in which physiological phenomena, the circulation of the blood, internal heat and respiration are ordered according to a higher organizing principle. Leonardo considered that the soul acts through the four humors, which ensure the balance and cohesion of the body. For Leonardo it is the seat of emotions and inner feeling, it is moreover from this ancient conception that French expressions such as *"avoir mal à l'âme"* which conveys profound emotional suffering, and by expression *"le vague à l'âme"*, which denotes a diffuse melancholy and inner yearning, both testifying to the ancient idea of a profound unity between body and mind. This thought is embodied, by Leonardo, in a quotation from the Codex Atlanticus, f.198r.:

«I movimenti dell'anima sono le cause delle varie complessioni degli uomini.»

«The movements of the soul are the cause of the various humors. »

Thus, Leonardo's dissections, beyond an anatomical description of the human body, also aimed to understand this invisible organization and the dynamic relations that unite material life, thought and the divine principle that orders them. Thereafter, Leonardo multiplied dissections in various hospital and academic contexts, in Florence, then in Milan (probably at the Ospedale Maggiore), and finally in Rome at the hospital of Santo Spirito in Sassia, where accusations of sacrilege put an end to his work[25], his methodical notes bear witness to a rigorously scientific approach ahead of its time, in Manuscript A, preserved at the Institute de France, he described in detail his working method, he explained that he had opened more than ten human bodies, proceeding *"layer by layer"*, removing successively the flesh, the tissues and the blood to reveal the veins, down to the finest capillary branches (figures 5), a single body did not suffice for him, he repeated the operation endlessly in order to attain knowledge as exact as possible.

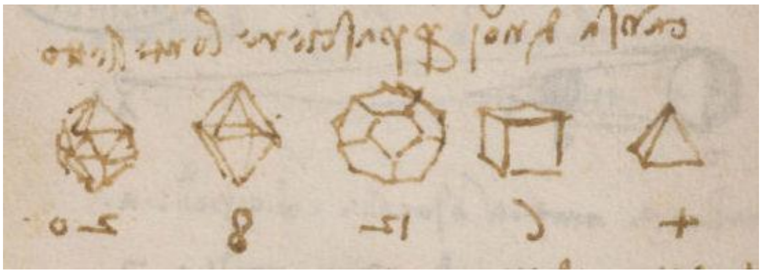

Figures 5. *Anatomical studies, RCIN 919001, Da Vinci*

Around 1510–1511, he collaborated in Pavia with the physician Marcantonio della Torre, a young professor of anatomy trained in Padua, together they planned to compose a great illustrated anatomical atlas, but the premature death of della Torre interrupted this undertaking[26]. Yet this abandoned project did not put an end to Leonardo's insatiable curiosity, his notebooks bear witness to a visionary mind driven by a constant quest to understand the laws of living beings.

After Hippocrates, Plato (427–347 BC), disciple of Socrates and founder of the academy of Athens, inherited the Pythagorean intuitions and the traditions of Empedocles and Hippocrates, who had conceived the world from the four fundamental elements (fire, air, water and earth), in the Timaeus[27] he gave them a geometric form, the tetrahedron for fire, the octahedron for air, the icosahedron for water and the cube for earth, to these four solids he added a fifth, the dodecahedron, which he associated with the universe as a whole (figures 6). The latter gathered the four others and united them into a superior totality, the dodecahedron thus became the figure of the One, first principle from which all things arose, Plato saw in these perfect forms the reflection of the rational order of the cosmos, for he thought that the universe had been geometrically designed by the Demiurge (the divine craftsman who shapes the material world from eternal forms). At the entrance to his school could be read the famous motto: *"Let no one ignorant of geometry enter here*[28]*",* an affirmation of the central role of geometry in the formation of the mind and in the understanding of reality.

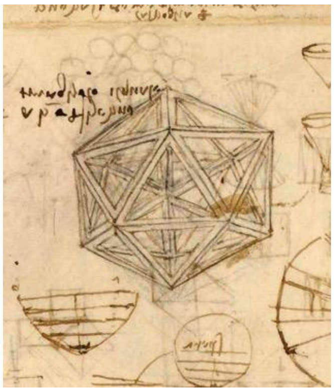
At the end of the 15th century, Leonardo da Vinci produced the illustrations for the treatise of his friend, the mathematician monk Luca Pacioli, in his work De Divina Proportione (1496–1498), published in 1509[29], devoted to the idea that a certain universal proportion present in the universe is found in the human body. Leonardo studied the regular and semi-regular polyhedra of Plato, in solid and transparent versions (figures 6). Among them, the dodecahedron occupied in his research a singular place, for it contains within it harmonic ratios, indeed, in a regular dodecahedron, each face is a pentagon, and in the latter, the ratio between the diagonal and the side is that of an ideal proportion, the golden ratio (≈1.618). Thus, this ratio is found at all scales of the figure, the Greeks recognized in the dodecahedron the geometric imprint of an intelligence ordering the world, that of the Demiurge[30]. Like Plato, Leonardo saw in mathematics the necessary foundation for all true knowledge, from the introduction to his Treatise on Painting, he reaffirmed this idea with a warning: *"Let no one read me who is not a mathematician[31]."*

At that time, as in the Renaissance, philosophy was not separated from mathematics, the philosopher was also a geometer, for understanding the world required knowing its ratios, its proportions and its measures. Rational thought, science and contemplation formed one and the same path toward truth, this is why the sages of Antiquity conceived the knowledge of number as a path to the divine. Little by little this domain moved away from the science from which it originated, to attach itself more to logic, ethics, language and consciousness.



The five polyhedra, Manuscript M, f 80v., Institut de France, DaVinci

Dodecahedron, Codex atlanticus Fol 707 r
Da Vinci



Icosahedron, Codex Atlanticus Fol 518 r.
Da Vinci

Figures 6

After Plato and his vision of a cosmos ordered by unity and geometry, his disciple Aristotle (384–322 BC) marked a turning point in the history of thought. Where Plato sought truth in the world of Ideas, that is, in an intelligible reality, Aristotle affirmed that it must first be found in sensible experience, that of observation and the study of phenomena accessible to the senses[32]. An encyclopedic philosopher, he was interested in all fields of knowledge, his method was based on the conviction that knowledge is born of observation and experience but must then rise toward universal principles.

Leonardo da Vinci has often been compared to Plato, because of his quest for harmony and unity of the world, central themes of Platonic philosophy. This comparison appears legitimate because Leonardo belongs to the humanist culture of the Renaissance, which rediscovered Plato and his ideal forms with the same conviction that the soul transcends the body. Plato, in the Phaedo[33], maintained that the body is nothing but a tomb that imprisons, and that only the soul leads to truth. Leonardo, several centuries later, continued this idea by writing in the Windsor Codex (Anatomical Manuscript A, fol. 2r):

«E tu, o omo, che consideri in questa mia fatica l'opere mirabili della natura, se giudicherai essere cosa nefanda il distruggerla, or pensa essere cosa nefandissima il tôrre la vita all'omo; del quale, se questa sua composizione ti pare di maraviglioso artifizio, pensa questa essere nulla rispetto all'anima che in tale architettura abita; e veramente, quale essa si sia, ella è cosa divina: sicché lasciala abitare nella sua opera a suo beneplacito…»

« And you, O man, who contemplate in this work how admirable the works of nature are, if you judge that it is an abominable thing to destroy them, think that it is infinitely more abominable to take the life of a man. For if this outward structure appears to you to be of marvelous craftsmanship, consider that it is nothing in comparison with the soul that dwells in such an architecture, and in truth, whatever it may be, this soul is a divine thing. Therefore, let it remain in its work according to its good pleasure… »

However, if Leonardo shared with Plato this quest for transcendence and unity, his method of research brings him closer to Aristotle, whom he cited on several occasions in his notebooks, sometimes to approve him, sometimes to contradict him. In his notes he explicitly referred to the philosopher when discussing the elements of nature and movement, this constant presence shows that Aristotle was for him a true authority of reference.

Leonardo shared with Aristotle the importance given to direct experience as the foundation of knowledge. In an epistemological approach he did not content himself with repeating what he read in books, he verified, experimented and constantly confronted received knowledge with the test of experience. Moreover, in his writings he strongly criticized those who limited themselves to repeating ancient authorities without verifying for themselves. This mistrust of abstract speculation and this requirement of verification through observation correspond directly to the Aristotelian method.

He wrote, in Manuscript G of the Institut de France, fol. 8r:

« Quelli, che s'innamoran di pratica sanza scienza, son come 'l nocchiere, ch'entra in navilio sanza timone o bussola, che mai ha certezza dove si vada. Sempre la pratica dev'esser edificata sopra la bona teorica; della quale la prospettiva è guida e porta, e, sanza questa, nulla si fa bene. »

« Those who fall in love with practice without science are like the pilot who boards a ship without rudder or compass and who never has certainty of the direction he takes. Practice must always be founded on a good theory, whose perspective is the guide and the gate. Without it nothing is done well. »

He added, in the Codex Atlanticus, folio 76 recto:

« Chi disputa allegando l'autorità, non adopra(lo) 'ngegno, ma più tosto la memoria. »

« He who, in disputing, invokes authority does not employ his intelligence, but rather his memory »

Like Aristotle, Leonardo devoted hundreds of pages to biology, to the study of animals and of the human body. His ambition was the same as that of the Greek philosopher, to transform empirical observation into rational science. Whether concerning the flight of birds, the horse or man, Leonardo followed a method rigorously founded on observation, description and comparison.

Leonardo compared the human skeleton and the equine skeleton, at the level of the legs and their proportions. This analogy directly extends Aristotle's assertion according to which the legs of the horse are like those of man[34] (figure 7). As an attentive observer, Leonardo resumed, verified and confirmed through drawing this ancient intuition.

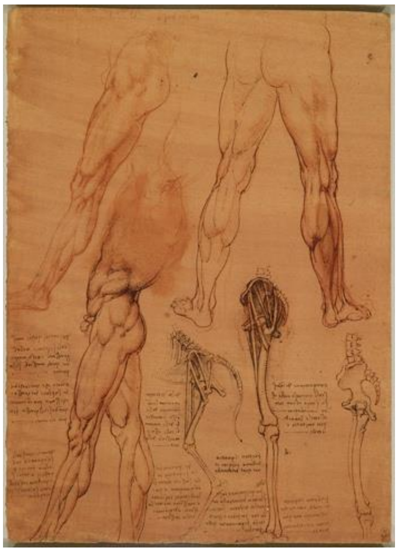


Figure 7. *RCIN 912625, Royal Collection Trust Codex Windsor, Da Vinci*

His scientific curiosity was also nourished by his reading. Leonardo possessed in his library[35] the Historia Naturalis of Pliny the Elder (23–79 AD), a true encyclopedia of the natural world, which he regarded as a work of reference. In it he found material for deepening his knowledge of minerals, plants, animals and the human body.

After Aristotle, Euclid (around 325–265 BC) brought a new form of rigor and clarity that would mark the entire history of science. A mathematician active in Alexandria, probably under the reign of Ptolemy I Soter, Euclid composed the Elements[36], a monumental treatise in thirteen books that gathers, organizes and systematizes the whole of the geometric knowledge of his time.

Euclid inherited from Thales, Pythagoras, Plato and Aristotle, but also from the more ancient knowledge that came from Egypt and Babylonia. However, he gave it a new form, founded on clear definitions, simple axioms and a succession of demonstrations. Each theorem followed logically from the preceding one, like the links of a perfect chain. This deductive method, of unprecedented rigor, would hold authority for more than two millennia.

What distinguishes Euclid is his ability to transform scattered discoveries into a coherent system. The Egyptians had practiced land surveying, the Babylonians numerical astronomy, the Pythagoreans the study of proportions, and Plato had given a cosmological dimension to geometric forms. Euclid took up this heritage and made it into a universal science. Geometry ceased to be a practical art or a philosophical speculation and always became a rational language applicable in all places.

Among the many notions presented in the Elements is the division of a segment *“in extreme and mean ratio”* (The Elements, Book VI, Def. 3). It is the first mathematical definition of the famous golden ratio (≈1.618). Euclid demonstrated that this ratio expressed a particular proportion already present in the dodecahedron composed of twelve regular pentagons.

This proportion, present in the structure of the regular pentagon, was no doubt known to the Pythagoreans, who saw in regular figures the expression of the order of the world. According to Proclus, in one of his commentaries on the first book of Euclid[37], Hippasus of Metapontum, a Pythagorean of the 5th century before our era, is said to have discovered the construction of the regular dodecahedron and the irrational ratios it contained. He reported: *« It is said that one of the Pythagoreans was the first to discover the dodecahedron in the sphere, and that having made public the way to inscribe it there, he disappeared under the action of the god, for it was just that he die, he who had revealed what ought to remain secret. »*

This tradition illustrates the discipline of silence (ἐχεμυθία) that reigned in the Pythagorean school, where all knowledge was held to be sacred. Mathematical discoveries were perceived there as revelations of divine order and their divulgation constituted a grave fault. At that time knowledge was not accessible to all, it belonged to a spiritual practice, a path of inner purification reserved for a restricted circle of initiates. As Jamblichus[14] reminds us: *« Among the Pythagoreans, all teaching had a sacred character reserved for initiates, the word of the master was a holy thing, and any doctrine revealed outside the circle of disciples was regarded as a profanation, those who violated the rule of silence and divulged the teachings were excluded from the community as impure. »*

Porphyry also confirmed the initiatory character and the secret pedagogy of the school: *« To those who came to listen to his teaching, he ordered five years of silence to test whether they knew how to control their tongue and be masters of their words, and if they passed this time in calm and gentleness, then he admitted them to the contemplation of the sciences. »*

These testimonies express the vertigo that may have been caused by one of the most overwhelming discoveries in the entire history of Greek thought. The uncovering of irrational numbers, revealed in the construction of the dodecahedron, called into question the very heart of the doctrine of Pythagoras, which rested on the idea that the whole universe obeys measure and number. Thus, by giving these figures a rational structure, Euclid transformed a mystical intuition into a true demonstrative system founded on measurement, axiom and proof.

This lineage, which marked the passage of Greek thought from an esoteric dimension to a more rational science, makes it possible to understand how the return to the spirit of antiquity made the heart of the Renaissance beat.

Euclid was rediscovered in the West as early as the 12th century thanks to the translations of Adelard of Bath (around 1080–around 1152) and of Campanus of Novara (around 1220–1296)[38], then printed in Venice in 1482. It was in this climate that Leonardo da Vinci (1452–1519) studied geometry. His codices bear witness to a constant interest in the same regularities as those brought to light by Fibonacci (1170–1250), author of the famous numerical sequence (1, 1, 2, 3, 5, 8, 13…), whose successive ratios tend toward the golden ratio[39] and express the harmony present in nature.

Leonardo filled his notebooks with drawings of helices, spirals and vortices (figures 8). This fascination is found in his studies preserved at the Royal Collection Trust, where he analyzed the movements of water in the form of whirlpools. Even in his visions of the deluge, as in his observations of storms and the movement of air, the raging waters organize themselves into spirals, revealing the omnipresence of these forms in nature.

Study of water
RCIN 912660, Codex Windsor, Da Vinci

Study of the Deluge
RCIN 912380, Codex Windsor, Da Vinci

Figures 8

This search for regularities also found an extension in his anatomical studies. Fascinated by the movement of fluids, Leonardo dissected ox hearts to better understand the shape of the ventricles and the movement of the blood. He injected wax into the cavities to obtain molds and visualize the internal structures. He observed that the blood flow inside the heart naturally organizes itself into a spiral, a movement that closes the cardiac valves rapidly and efficiently (figure 9). His observations, which he described in his anatomical notes on the heart (RL 19073r), bear witness to a remarkable understanding of the role of natural geometry before modern science confirmed it.

Figure 9. *Study of the aortic valve*
Rcin 919083, Codex Windsor, Da Vinci

Following Euclid, who had given geometry its most rigorous form, Greek thought discovered with Archimedes (287–212 BC) the importance of applying mathematics to the understanding of forces, weights and movements. Born in Syracuse and probably, according to the philosopher Proclus[37], trained among the scholars of Alexandria, Archimedes was at once a mathematician, physicist, engineer and inventor. He embodies the ideal of the complete scholar, capable of linking theory to practice and placing mathematics at the heart of daily life.

His research on mechanics and balances was decisive. He established the laws of the lever and defined the notion of center of gravity, opening the way to a true science of forces. He also developed an experimental hydraulic knowledge, designing the famous Archimedean screw which made it possible to raise water (figure 10). He devised siege machines, catapults, pulleys and cranes inspired by nature and which reveal a rational understanding of the laws of physics. Through this ability to translate mathematics into concrete applications, he became one of the most powerful models of Antiquity.

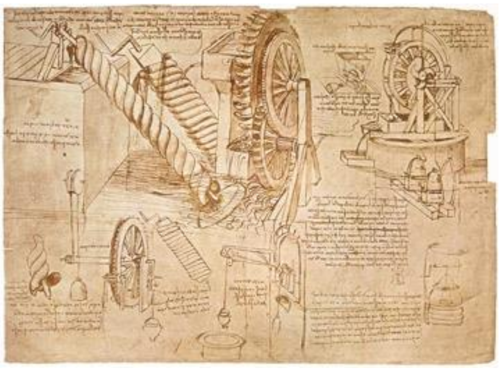


Figure 10. *Tool and machine, Archimedean screw*
F 386 r, Codex Atlanticus, Da Vinci

In his mechanical and engineering studies, Leonardo mentioned Archimedes on several occasions. Like his illustrious predecessor, he never separated theory from experience. His sketches of bridges, pulleys, pumps, hydraulic screws and war machines directly extend the Archimedean inspiration, in a constant search for the fundamental principles that govern movement and equilibrium.

A drawing by Leonardo da Vinci illustrates a reflection on the proportional ratios necessary for the equilibrium of mechanisms (figure 11). The figure is composed of two wheels of different diameters in a ratio of 5/4, the larger one has a diameter greater by a quarter than that of the smaller one. The two wheels are connected by a system of articulated connecting rods fixed to the hubs, forming two right triangles. This device belongs to a kinematic study and aims to understand the movement of the parts with respect to one another independently of the forces that produce them. The whole constitutes a mechanism based on the principle of the compound lever, where several levers cooperate to transmit, transform or amplify a movement. In this device the oblique bars and the axes fulfil the function of levers, ensuring the transmission of movement from one wheel to the other according to precise length ratios.

Figure 11. *Studies in mechanics*
Institut de France, Manuscript B, 36 recto

If the segments are extended to the base of the circles, the whole defines a rectangle whose sides are in a ratio of 3/2. This ratio is found in the transmission of forces according to unequal arms, comparable to the law of the lever formulated by Archimedes. It appears in the distribution of loads in a balanced system. The rear wheel supports about sixty per cent of the total weight, while the front wheel carries forty per cent. This arrangement ensures the stability of the whole with a center of gravity slightly displaced towards the rear. It should finally be noted that the two ratios present in this mechanism reflect the harmonic ratios described in the musical theory of Pythagoras, that of the major third (5/4) and that of the perfect fifth (3/2).

Thus, Leonardo linked the laws of the mechanics of Archimedes with the principles of musical harmony described by Pythagoras by inscribing them in elementary geometric forms (square, triangle, rectangle and circle). Like Archimedes, Leonardo saw in mechanics the living demonstration of the power of mathematics. In the manuscript E of the French Institute folio 8.v, he expressed this idea with clarity :

« La meccanica è il paradiso delle scienze matematiche, perché con quella si viene al frutto delle scienze matematiche. »

« Mechanics is the paradise of the mathematical sciences, for it is through it that mathematics bears fruit »

1.3. *Roman Antiquity, the transmission and the legacy of Greek knowledge*

From Greece to Rome, the ideal of a world ordered by mathematics was transmitted without rupture. Archimedes had shown its mechanical power, the architect and engineer Marcus Vitruvius Pollio, known as Vitruvius (1st century BC), made it the rule of the art of building. In his treatise De Architectura[40], the only architectural work of Antiquity that has come down to us, he took up the Greek heritage by integrating into it a profoundly unitary conception of the world. For him architecture had to reflect the order of the universe, and man constituted its model through his proportions which embodied both balance and harmony. Vitruvius thus affirmed that *« man is the measure of all things. »* Architecture therefore belonged to a cosmology in which man reflects the universe as a whole, an idea already present in Empedocles, Hippocrates or Plato, but which Vitruvius applied for the first time to the art of building.

In continuity with the Greek scholars, Vitruvius founded his theory of construction on the equilibrium of the four elements, earth, water, air and fire, whose proper combination ensures the stability and durability of structures. Earth, through its mass and cohesion, constituted the base of foundations and load-bearing walls, the Roman architect insisted on the quality of the soil, which must be tested before any construction (De Architectura, II, 1). Water, principle of life and union of materials, which intervenes in the preparation of lime, mortar and coatings, had to be pure, light and non-stagnant to guarantee the solidity of buildings (VIII, 3). For air, element of vital breath, he recommended choosing the orientation of cities and houses according to prevailing winds to preserve the health of inhabitants (I, 4). As for fire, Vitruvius advised charring the surface of wood to make it more resistant to humidity and insects (II, 9,2). Thus, building amounted to finding the right measure, that which maintains harmony between natural forces, where each element acts in correspondence with the others, forming an indivisible whole.

Within this conceptual framework, Leonardo da Vinci approached nature as a field of experimentation governed by the laws of physics. For him understanding the world meant observing its elementary forces, analyzing their effects and measuring their ratios. He studied earth through the structure of soils and rocks, noting the effects of time, erosion and gravity on the form of the landscape. He devoted countless observations to water, analyzing it as a driving force of the world. He measured its pressure, its speed and its turbulence, and studied the behavior of currents in rivers and canals. He explored the physics of air, observing resistance, lift and movement to understand the principles of flight and the propagation of sound. As for fire, he examined its effects on metals, pigments and wood. In folio 91 v. of the Codex Madrid I, Leonardo explicitly took up a piece of advice inherited from Vitruvius: *« Ligna quae leviter amburuntur, diuturniora fiunt. »* (II, 9,2)

« Il legno, la cui superficie sia stata leggermente abbrusciata, diventa più duro e più atto a resistere all'umidità. »

« Wood whose surface has been slightly burned becomes harder and more resistant to moisture »

This technique[41] has been identified on piles and various wooden elements uncovered at Ostia, whose remains date mainly from the 1st to the 3rd century AD, as well as at Pompeii and Herculaneum, preserved in their state of 79 AD, and in the military camps along the Germanic Limes occupied between the 1st and the 3rd century. The traces documented concern supporting piles and structural components as well as posts from military palisades and fence stakes. Taken together, this evidence demonstrates that the Romans already made use of superficial wood carbonization in a range of contexts, long before Leonardo da Vinci reformulated the principle during the Renaissance.

Leonardo da Vinci also extended the Vitruvian principles by applying them to the study of the human body. For him the same laws that govern matter are expressed in the proportions of man. He saw in the human body a naturally balanced system governed by the same physical laws as those of the world, gravity, tension, support and resistance[42]. The mechanics of the body constituted in his eyes a perfect model of efficiency from which he drew inspiration in his research and his inventions. In observing the functioning of the muscles, he transposed the human gesture into articulated systems (figure 12), and he took up the proportional ratios of the body in several of his creations, notably in the conception of his ideal horse.

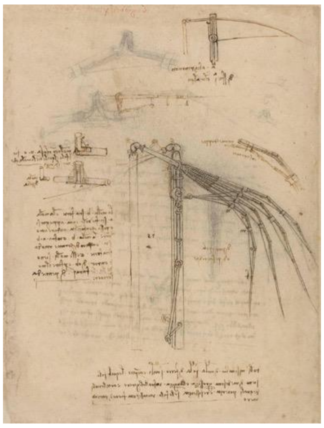


Figure 12. *Codex atlanticus f844r, Da Vinci*

2. Vitruvian Man according to Leonardo da Vinci

2.1. *The genesis and the sources of the drawing*

The most striking proof of the influence of ancient thought on Leonardo da Vinci remains his famous drawing of the Vitruvian Man. It concentrates in a single image an entire millennial heritage. In it we find the inspiration of Thales, one of the first Greek thinkers to have introduced geometric reasoning into the study of the world. In this drawing the human body is considered as a measurable figure, subject to geometry like any natural object. To this first intuition is added the influence of Pythagoras, through the embodiment of a visual harmony and a quest for perfection in which geometry becomes the means of approaching the ideal. But this harmony is not only that of the body, it is the expression of a vaster unity. This is where the inspiration of Plato comes in, who affirmed in the Timaeus[27] that “all comes from the One.” Leonardo illustrates man as a microcosm, a part integrated within a larger whole, reflection of a fundamental unity that links all things. The influence of Aristotle also appears through the empirical method founded on observation and experimentation that Leonardo used to conceive this drawing. This quest for perfection naturally leads to Euclid. The entire balance in the conception of this drawing rests on the harmony of geometry. To this is added the heritage of Archimedes, who revealed the laws of weight and equilibrium. The Vitruvian Man is represented in two superimposed movements which demonstrate that when the legs are spread and the arms extended at three quarters, man preserves his equilibrium and stability. On the other hand, with the legs together and the arms in the same position, this equilibrium would be lost. It is maintained, however, with the arms extended horizontally. This drawing thus illustrates, in a rational manner, the laws of gravity and stability of the human body.

However, it must be recalled that the genesis and representation of the Vitruvian Man were not born from Leonardo’s imagination. They are based on and respond to a problem formulated by Vitruvius in his treatise De architectura (Book III, chap. 1), in which the ideal proportions of the human body are presented. He wrote:

“The navel is, by nature, the center of the human body, for if a man lies on his back, with hands and feet extended, and one places a compass upon his navel, in tracing a circle one will pass through the extremities of the hands and feet, likewise the height of the man is equal to the span of his arms; thus a square can be traced around him.”

Vitruvius thus posed the basis of a reflection: how can a man be inscribed in two positions, both in a square and in a circle?

This challenge posed by the Roman architect was for a long time a formidable enigma for many scholars and artists. Mariano di Jacopo, known as Taccola (1382–1453)[43], sketched a schematic figure without anatomical precision (figure 13). Francesco di Giorgio Martini (1439–1501)[44] in turn attempted to reconcile geometry with the proportions of the body, but his drawings remained clumsy (figure 14). Finally, Giacomo Andrea de Ferrara (around 1450–1500)[45], attentive reader of Vitruvius and close to

Leonardo, proposed a still unstable solution in which the figure did not respect human anatomy (figure 15). All encountered contradictions or approximations. This shows how much this problem exercised a tenacious fascination while resisting the most skilled. In taking up this challenge, Leonardo was the first to give this ancient problem a visual solution both rigorous and universal. He transformed an architectural theory into a true geometric construction founded on the measurement of man.

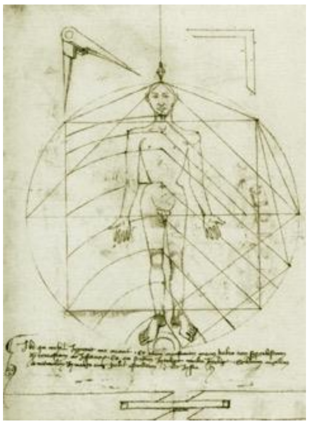


Figure 13. *Taccola, circa 1419/1450*

Figure 14. *Giorgio Martini, circa 1475/1482*

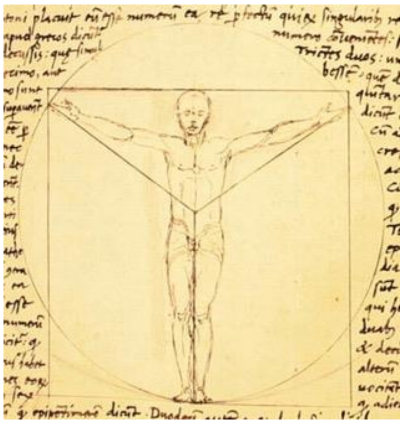


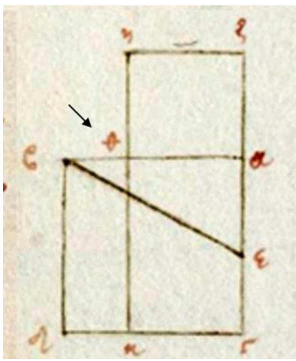
Figure 15. *Giacomo Andrea de Ferrara, circa 1490*

2.2. *Between infinity and measure*

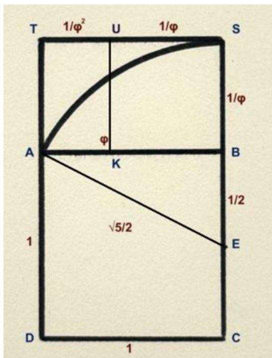
In this quest for unity, Leonardo belonged to the tradition of the Greek geometers, notably Pythagoras and Euclid, for whom beauty resulted from measure and just ratios. He saw in geometry a means of linking man to the universe. The human body, through its proportions, had to reflect a universal order at work in nature and in living beings, but also in the unfolding of forms, sounds and physical laws. It is in this continuity that Leonardo took an interest in the works of Euclid, whose Elements lay the foundations of classical geometry.

In Euclid, the search for ideal proportions rests on the idea of a balance between two ratios. This relation which links measure to harmony became a fundamental principle of geometry. The proportion called *« extreme and mean »* described in Book VI of Euclid's Elements offers the most complete example (figures 16). It consists in dividing a segment in such a way that the ratio of the whole to the greater part is equal to the ratio of this greater part to the smaller one. That is, if one divides a segment AB at a point C, one obtains the relation:

AB/BC = BC/AC



Euclidis Elementa, Vaticanus Graecus 190 Apostolic library of the Vatican, 9th century, folio 31r, section of the book VI.



Reproduction of the golden rectangle

Figures 16

To designate this harmony, Euclid used a distinctive sign represented by a circle crossed by a horizontal line. He placed it at the intersection between the height common to the square and to the small golden rectangle, at this point the ratio between these two segments is equal to the golden number because the side of the small golden rectangle is $1/\varphi$ since $EA = ES= \sqrt{5}/2$ et $SB= (\sqrt{5}-1)/2= 1/\varphi$ (figures 16).

In Greek thought, the circle represented unity, totality and perfection, as for the bar, in Greek geometric practice it indicated the rational act of dividing a segment, in other words the measurement of the whole by the part. The union of the circle and the bar thus embodies the Greek spirit of an ordered measure, where unity is divided into proportional parts.

In the nineteenth century, the American mathematician Mark Barr[46] proposed using the Greek letter φ to designate the golden number. According to an explanation often repeated in the literature, notably in Matila Ghyka[47] and other authors of the twentieth century, Barr is said to have chosen this letter in homage to the sculptor Phidias, to whom the use of the golden number was attributed. However, no direct proof confirms this hypothesis, it is most likely a later interpretation, one may therefore also suppose that familiar with the writings of Euclid, he reused the same sign but vertically, in continuity with the ancient sign.

Leonardo da Vinci was inspired by the studies carried out by Euclid (figure 17), even copying certain figures, among them an illustration of the Pythagorean theorem. In the figure made by Euclid the sign of a circle crossed by a horizontal line appears again (figure 18). This comparison has led to a reexamination of the significance of this figure.

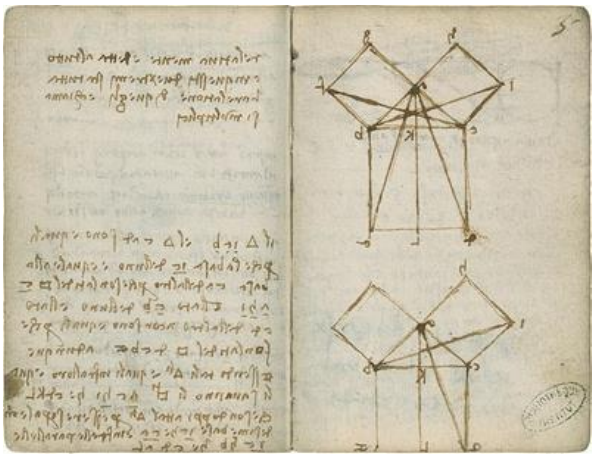


Figure 17. *Geometrics studies*
Da Vinci, Codex Arundel, British Library

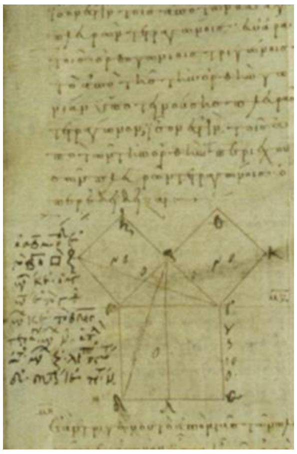


Figure 18. *Pythagorean theorem*
Element I, proposition 47
Apostolic library of Vatican

The analysis of the figure enabled us to discover the existence of an underlying geometric structure which links the Pythagorean theorem to the golden number (figure 19). If one considers a square ABCD of side 1 and one draws an arc of a circle with center E, situated at the middle of segment BC, and of radius $\sqrt{5}/2$, this arc makes it possible to construct a golden rectangle TSCD of side $(1+\sqrt{5})/2$, which passes precisely through the vertices O and L of the small Pythagorean square of side $(1/\sqrt{2})$ because its diagonal BR is equal to 1. The point of intersection M between the golden rectangle TSDC and the small Pythagorean square determines the position of the golden rectangle MPSB as well as that of the square TMPA. In an analogous manner, by extending segment OB (side of the small Pythagorean square) to segment TS one obtains point U, which makes it possible to define a golden rectangle TUKA as well as a square USBK.

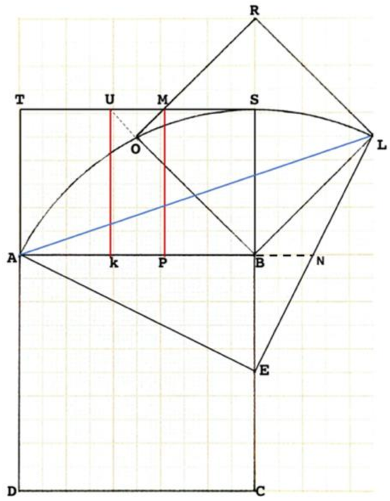


Figure 19. *Reproduction of golden rectangle with Pythagorean theorem*

L is situated 1/2 to the right of the large square of side 1 because OL=1. N is the midpoint of LE, situated on the line passing through AB. Thus BN = 1/4 and AE = EL = $\sqrt{5}/2$. AEL is therefore a right triangle.

By the Pythagorean theorem:

$$AL = \sqrt{(AE^2 + EL^2)} = \sqrt{(5/4 + 5/4)} = (\sqrt{10})/2$$

The area of the square of side AL, associated with angle AEL, is then equal to the sum of the areas of the golden rectangle TSCD, of the small Pythagorean square ORLB and of the square USBK or TMPA.

Thus, $AL^2 = (TS \times SC) + OB^2 + US^2$ where $US = SC - 1 = 1/\varphi$

$$(\sqrt{10}/2)^2 = (1 \times \varphi) + (1/\sqrt{2})^2 + (1/\varphi)^2$$

These ratios can be explained by the length $\sqrt{5}/2$, which acts as a genuine geometric hinge between the golden rectangle and the Pythagorean theorem. This length serves as a multiplicative factor within the family of associated right triangles (Figure 20).

Indeed, if in a right triangle the largest side is twice the smallest, then the area of the triangle is equal to the area of the square constructed on the smallest side. This process continues by taking half of the hypotenuse as the new smallest side. One observes that the smallest side has gone from 1/2 to $\sqrt{5}/4$ and has therefore been multiplied by $\sqrt{5}/2$. Likewise, the area has increased from 1/4 to 5/16, meaning it has been multiplied by $(\sqrt{5}/2)^2$.

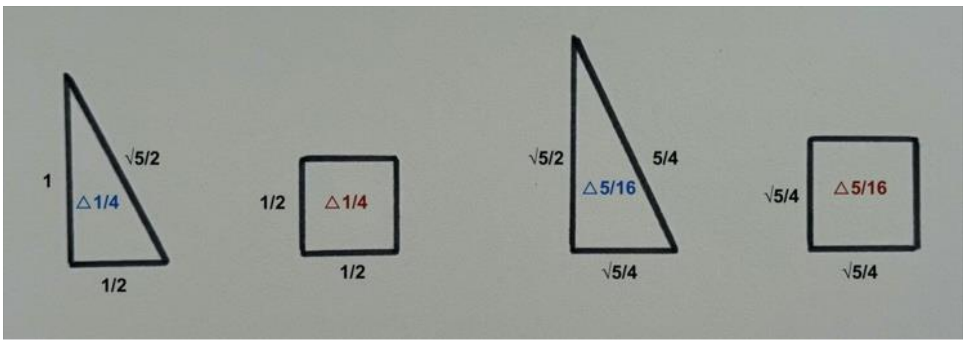


Figure 20. *Geometric representation of the sequence of right triangles governed by the √5/2 progression*

This construction also reveals a new whole recurrent sequence defined geometrically from an initial square and an iteration of the Pythagorean theorem. The successive ratios converge toward the proportion of the golden number. The values of the sequence are: 1, 4, 5, 9, 14, 23, 37, 60, 97, 157… This progression follows a rule of addition in which each term is the sum of the two preceding ones.

The first term corresponds to a small square of side 1/2, the area of this square is therefore $(1/2)^2 = 0.25$. To avoid fractions one chooses a new scale in which the area 0.25 becomes 1. The second term is a square of side 1 which becomes 4. In this new scale, the third comes from the hypotenuse of the right triangle of the previous values. Its square of area 1.25 becomes 5 (1 + 4 = 5).

Thus, at each step, a right triangle is formed from the preceding lengths, then a new square is constructed on the hypotenuse obtained. This geometric growth generates a series of successive ratios that tend toward a limit value corresponding to the square root of the golden number.

Let R(k) denote the ratio between two successive lengths. The following table presents these values in exact and decimal form, which illustrates the convergence toward $\sqrt{\varphi} \approx 1.272$ (figure 21).

Indice K	R(k)=√(n(k+1) / n(k))	(R(K) (Decimal value)
1	√(5/4)	≈ 1.118
2	√(9/5)	≈ 1.341
3	√(14/9)	≈ 1.247
4	√(23/14)	≈ 1.281
5	√(37/23)	≈ 1.269
6	√(60/37)	≈ 1.273
7	√(97/60)	≈ 1.271
8	√(157/97)	≈ 1.272
9	√(254/157)	≈ 1.271
10	√(411/254)	≈ 1.272
11	√(665/411)	≈ 1.272
12	√(1076/665)	≈ 1.272
13	√(1741/1076)	≈ 1.272
14	√(2817/1741)	≈ 1.272
15	√(4558/2817)	≈ 1.272

Figure 21. *Table of ratios R(k)*

The numerical study shows that from the fifth term onwards, the difference between two successive ratios decreases according to a geometric progression with ratio ($1/\varphi \approx 0.618$). This property is well known in the analysis of recurrent sequences. However it here takes an unprecedented form, applied to a geometric construction derived from the Pythagorean theorem (figure 22).

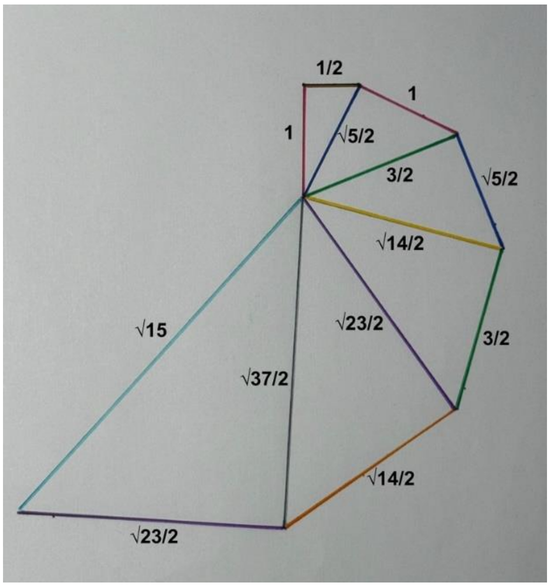
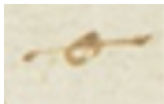


Figure 22. *Geometric representation of the sequence that converges toward √ φ*

2.3. *The unity of the world*

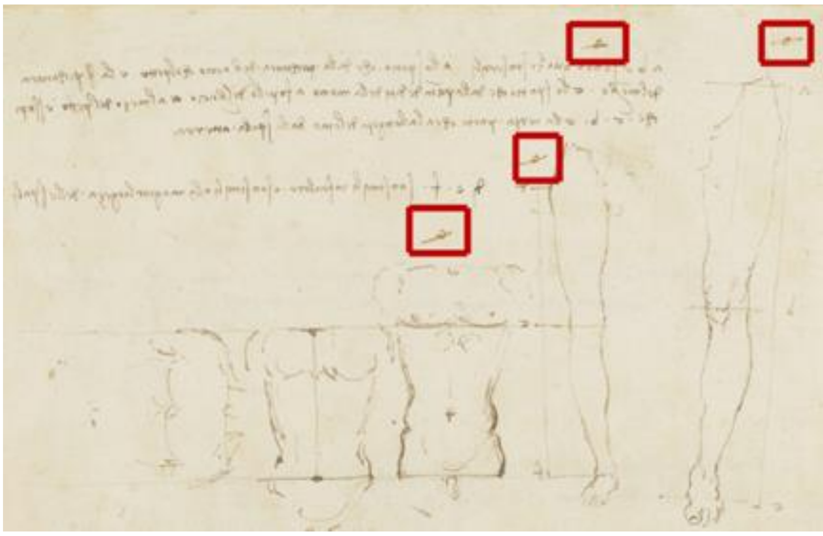
The idea of a harmony between measure and infinity, inherited from the Pythagorean tradition[48], profoundly marked the thought of the Renaissance. Leonardo da Vinci, drawing upon ancient texts, sought to understand and to reveal the fundamental ratios that structure the universe. In his studies on the proportions of the body Leonardo explored the measurable ratios that structure human anatomy according to principles of harmony and proportion[49]. He noted these ratios in the form of fractions. It was not for him a matter of rediscovering the golden number as an absolute value, but rather of understanding how concrete and rational ratios could express the harmonic order of the world. To express the equilibrium of geometric ratios, of a measure capable of uniting the multiple and the totality, he took up the same symbol as Euclid (figures 23, 25).

Euclide



Da Vinci

Symbol of harmony



Figures 23. *Studies of proportions,*
Rcin 919130, Royal Collection Trust, Da Vinci

In the conception of the Vitruvian Man (figure 24), Leonardo da Vinci copied onto the drawing the proportions established by Vitruvius, but he did not limit himself to reproducing them. He made them a principle of experimentation. Through direct observation of the body, through dissection and the study of movement, he verified these ratios, corrected them and harmonized them so that each part contributes to the equilibrium of the whole. His objective went beyond measurement, he sought an organic unity in which the human form is no longer reduced to a sum of segments but is understood as a system of correspondences.

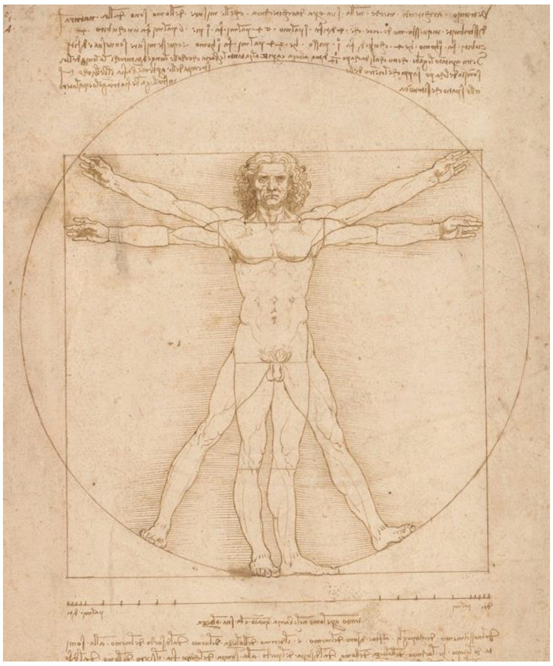


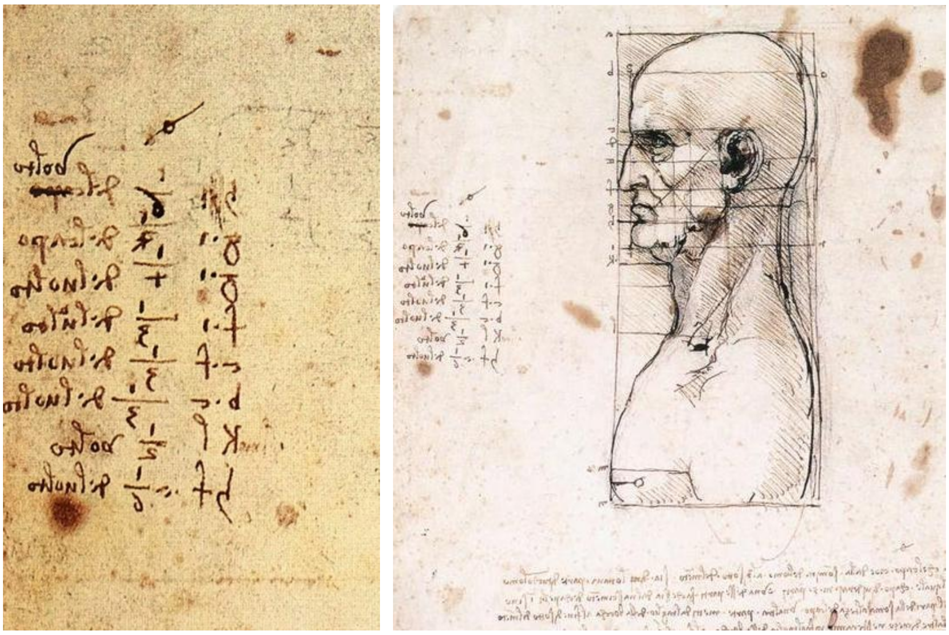
Figure 24. *Vitruvian Man,*
Gallerie dell'academia of Venise, Da Vinci

Leonardo observed that the different parts of the body correspond to one another according to a rigorous system of proportional ratios (figures 24, 25). The width of the shoulders is equal to half a leg, a proportion identical to that which runs from the torso to the pubis or from the torso to the top of the head, the hand, for its part, corresponds to the dimensions of the face, from the root of the hair to the chin, as well as to that which separates the navel from the pubis. The length of the foot is equivalent to the distance from the wrist to the elbow but also to that of the torso to the navel.

Many parts of the body have a length double one another, for example the length from the pubis to the soles of the feet corresponds to half the height of the body. The height between the neck and the pubis is twice that between the neck and the top of the head, and the distance from the neck to the top of the head is in turn twice that from the neck to the torso, the same logic is observed in the upper limbs, the distance from the shoulder to the middle of the torso is twice smaller than that from the middle of the torso to the elbow, and the length from the elbow to the hand is found doubled in the total reach of the arm to the middle of the torso.

Other ratios complete this structure according to regular multiplications. The segment between the torso and the pubis is three times larger than that which separates the neck from the torso, the height from the pubis to the top of the head presents the same ratio with the distance from the neck to the top of the head. The height from the neck to the pubis is in turn four times larger than that from the neck to the torso, while the distance from the pubis to the soles of the feet represents six times its length.

This principle is found in his studies of human proportions, notably in a drawing preserved at the Gallerie dell'Accademia of Venice, where Leonardo segmented the face of a man in profile. On the left side of the sheet appear the fractions 1/2, 1/3, 1/4 and 1/6 (figure 25).



Figures 25. *Studies of proportions*
Gallerie dell'académia of Venise
Da Vinci

Each measurement that refers to another reveals a principle of self-similarity. The body of the Vitruvian Man manifests a geometric recurrence, as if the human form carried within it a mathematical message. Thus, each part becomes a multiple or a rational submultiple of the whole.

The multiples of 2, 3, 4 and 6 employed by Leonardo reveal that the structure of the body rests upon a duodecimal system[50] inherited from ancient methods of measurement. In Antiquity base 12 was considered the most harmonious because it made it possible to generate many simple and exact divisions, the half, the third, the quarter, the sixth. It offered a flexibility of calculation which the decimal system could not attain.

In the Renaissance as well, this system formed part of daily life. In currency it structured all values, twelve deniers formed one sou, twenty sous were equivalent to one livre, that is two hundred and forty deniers for one livre. Large unit of one hundred and twenty units were also used, as they were easier to divide into dozens for commercial purposes. The same principle was applied to lengths and weights, were twelve inches made a foot and twelve ounces one pound. This model, inherited from Rome, was still taught by Luca Pacioli in his Summa de arithmetica[51] (1494), in which he described merchant calculation methods founded on these divisions into twelve.

Vitruvius himself made use of this metrological principle in the directives he formulated to establish the proportions of the ideal man[40]: *"Four palms make a foot, six palms make a cubit and twenty-four palms make a man."*

However, he did not always respect the duodecimal system in his instructions. He affirmed for example that the head had to represent one tenth of the total height of the body, which introduces a decimal logic foreign to ancient division.

Leonardo da Vinci therefore took up the statement of Vitruvius, which he copied almost word for word on his famous drawing of the Vitruvian Man. However, he did not follow him in his measurements. If he adopted the duodecimal system inherited from the ancient tradition, as shown by the ratios founded on the multiples of 2, 3, 4 and 6, the proportions he observed show that this model did not suffice to account for the complexity of the human body.

For certain parts such as the foot, the hand or the face he extended the duodecimal principle to a finer scale founded on a common base of 120 divisions of the unit, the smallest common multiple of the principal denominators employed to describe the ratios of the human form (figure 26).

Body part	Value	Exact fraction
From the top of the head to the soles of the feet	1	1/1
From the navel to the soles of the feet	0,60833	73/120
From the pubis to the soles of the feet	0,5	1/2
From the tip of the hand to the middle of the torso	0,5	1/2
From pubis to the top of the head	0,5	1/2
From the top of the head to the navel	0,3916	47/120
From the neck to the pubis	0,3333	1/3
From the torso to the pubis	0,25	1/4
Width of the torso (from one shoulder to the other)	0,25	1/4
From the top of the head to the torso	0,25	1/4
From the knee to the soles of the feet	0,25	1/4
From the pubis to the knee	0,25	1/4
From the middle of the toroso to the elbow	0,25	1/4
From the elbow to the tip to the hand	0,25	1/4
From the neck to the top of the head	0,1666	1/6
Lenght of the foot	0,1416	17/120
From the wrist to the elbow	0,1416	17/120
From the torso to the navel	0,1416	17/120
From the shoulder to the middle of the torso	0,125	1/8
From the navel to the pubis	0,108	13/120
From the hairline to the chin (face)	0,108	13/120
From the tip of the hand to the wrist (hand)	0,108	13/120
From the neck to the torso	0,0833	1/12

Figure 26. *A table of the proportions as they can be inferred from Leonardo da Vinci's Vitruvian Man*

The choice of a base of 120 units makes it possible to obtain ratios that become perfectly measurable (40, 47, 60, etc.). For example, half of the body corresponds to 60 units out of 120, and the length of the foot to 17 units, that is a proportion of 17/120. Thus, each value of the sequence is calculated from the previous one according to a simple and rational ratio, which ensures the continuity and harmony of the system of proportions (figure 27).

Ratio	Decimal value	Equivalent over 120 divisions
1 (whole body)	1	120
73/120	0,60833	73
1/2	0,5	60
47/120	0,3916	47
1/3	0,33333	40
1/4	0,25	30
1/6	0,16666	20
17/120	0,14166	17
1/8	0,125	15
13/120	0,108	13
1/12	0,08333	10

Figure 27. *Equivalence of ratios on a base of 120 divisions*

The number 120 is also part of the long ancient tradition of research on the harmony of proportions, within the sexagesimal system[4] inherited from the Mesopotamian civilizations to define the measurement of time and space. Leonardo's choice to adopt this numerical base reveals his desire to anchor the science of the human body in the continuity of an ancient knowledge in which measurement and proportion express the universal order. Moreover, the interest of this system also lies in its harmonic flexibility, it makes it possible to generate and combine both binary ratios (1/2, 1/4, 1/8) and ternary ratios (1/3, 1/6, 1/12) within a single structure of measurement. Applied to the proportions of the human body it acts as a *"harmonic modulus"*, that is a reference unit which links all the subdivisions of the body to a common scale.

The radius of the circle whose center is the navel has a value (73/120), close to the inverse of the golden number (0.618). Consequently, the figure seems to be inscribed in two superimposed golden rectangles, one arising from the geometry of the square, the other from that of the circle.

Beyond its arithmetic value the number 120 occupies, in the Pythagorean tradition[52], a structuring place. It is interpreted as a principle of numerical organization which reflects the harmony of the world. Among the thinkers of Antiquity[53] mathematical ratios were perceived as models of universal order capable of accounting both for the movement of the stars and for the structures of living beings.

The number 120 results from the product of the first five integers ($1 \times 2 \times 3 \times 4 \times 5 = 120$). This represented for the Pythagoreans a hierarchical conception of the world which symbolized the progressive passage from unity to complexity, from abstraction to incarnation. Each number was thus understood as a stage that participates in the genesis of reality.

In Greek thought this gradation is expressed through five levels[54], the Monad (1), principle of unity and origin. The Dyad (2), principle of duality and differentiation. The Triad (3), principle of harmony and mediation. The Tetrad (4), principle of organization and stability of the world. And the Pentad (5), principle of generation and life (figure 28). The whole (1 to 5) defines a complete structure in which the combination of these numerical ratios is perceived as the basis of every organized form.

Number	Pythagorean Symbol	Meaning
1	Monad	Divine principle, absolute unity, origin of all being.
2	Dyad	Duality, polarity, separation of principle and matter.
3	Triad	Harmony, mediation, equilibrium between opposites.
4	Tetrad	Structure of the world, stability, the four elements.
5	Pentad	Life, generation, humankind as microcosm of the whole.

Figure 28. *The Pythagorean hierarchy of numbers*

In Plato, notably in the Timaeus[27], where the five perfect solids constitute the fundamental geometric structure of the cosmos, the dodecahedron, symbol of unity, and the icosahedron, mirror of the former, possess a complete symmetry group comprising one hundred and twenty transformations, sixty rotations and sixty reflections, which return the figure to itself without altering its structure (figures 6). This number 120 illustrates both the harmonious complexity of these figures and the geometric perfection that the philosophers of Antiquity associated with the universal order.

In later monotheistic traditions the number 120 also retains a symbolic value. In Genesis (Old Testament, 6, verse 3) God fixed at one hundred and twenty years the duration of human life. In the Book of the Acts of the Apostles (New Testament, 1, verse 15) this same number appears again. One hundred and twenty disciples are gathered at the descent of the Holy Spirit. In early Islamic tradition the duration of one hundred and twenty days corresponds to the time necessary for the formation of the human being. Embryonic development is described there as unfolding in three phases of forty days each, first in the form of a drop, then of a clot, and finally of an embryo (Bad'al-Khalq, hadith n°3208). Thus, from Pythagorean arithmetic to theology this number appears as a unit of structure and totality which links the science of proportions to a global conception of the order of the world.

This relation, which rests on a search for equilibrium between the multiple and unity, Leonardo da Vinci also expresses in his observations on plant growth (figure 29). He noted in the Codex Arundel, folio 152 r.:

« Ogni anno, quando i rami d'un albero hanno compiuto la loro crescita, la somma delle loro grossezze, in ogni livello di ramificazione, è sempre eguale a quella del tronco. »

« Each year, when the branches of a tree have completed their growth, they will together have a thickness equal to that of the trunk, and at each level of branching the thickness of the branches issuing from the same division will always be equal to that of the trunk. »

Through this remark Leonardo transposed a law of conservation[55] in which, at each level of division, the sum of the diameters of the daughter branches is equal to the diameter of the trunk. In other words matter divides but unity is maintained.

Figure 29. *Study of the tree's proportion*
Da Vinci, Codex Arundel, British Library

This principle admits a direct parallel in Pythagorean music[16], where the series of twelve fifths converges toward unity. To illustrate this correspondence, the Pythagoreans expressed harmonic ratios in terms of frequencies. They observed that the perfect fifth, fundamental interval of the scale, is defined by a ratio of 3/2 between two sounds.

Let f be the frequency of a fundamental note. The perfect fifth corresponds to the ratio 3/2. The note located a fifth above has for frequency:

$f(\text{quinte}) = (3/2) \times f.$

By chaining twelve successive perfect fifths, that is twelve intervals of ratio 3/2 applied one after another, one obtains the twelve notes of the musical system which forms the complete cycle of sounds. The starting frequency is therefore multiplied twelve times by 3/2, which gives:

$f(12 \text{ quintes}) = f \times (3/2)^{12}.$

However, after the addition of these twelve fifths, the pitch obtained exceeds the zone where the initial note was situated. The result is found approximately seven octaves higher, since twelve fifths cover roughly seven octaves. In music an octave corresponds to an interval whose frequency is double the previous one (ratio 2/1). Thus, each octave multiplies the frequency by 2, and seven octaves correspond to a multiplication by 2^7.

To bring this frequency back into the same sound register, that is into the same range of pitch as the starting note, it is therefore necessary to divide the frequency obtained after the twelve fifths by 2^7:

f(normalized) = f(12 quintes) / 2^7.

The normalized value obtained corresponds to the initial note replaced in its original register. The ratio is:

$(3/2)^{12} / 2^7 \approx 1{,}0136$.

This result is very close to 1, which shows that the series of twelve fifths returns practically to the initial note. Thus, from the proportion of the human body to that of music and of nature, the same principle of unity is manifested.

Leonardo da Vinci conceived nature as a divine work governed by laws of absolute perfection. A true spirit of the Renaissance, he did not seek to oppose faith and reason, but to unite them in a single quest for truth. In his writings, following Plato who said: *« God is the measure of all things, far more than man ever was, as Protagoras says*[56] », Leonardo affirmed that natural forces are only the instruments of the divine will.

This conviction appears in the note of the Codex Arundel, f 155v, British Library:

«O mirabile necessità! Tu costringi ogni effetto ad essere il risultato di sua causa; e per una legge breve, severa e immutabile, disponi ogni azione naturale.»

«O marvelous necessity! You constrain every effect to be the result of its cause, and by a brief, severe and immutable law you dispose of every natural action. »

By these words Leonardo proclaimed that the movement of the cosmos participates in divine harmony.

In this same perspective he noted in the Codex Atlanticus, F 273r, Biblioteca Ambrosiana:

«Io t'ubbidisco, Signore, prima per l'amore che ragionelemente portare ti debbo, secondaria che tu sai abbreviare o prolungare le vite a li omini. »

« I obey you, Lord, first through the love that I reasonably owe you, then because you alone can shorten or prolong the lives of men. »

These words reveal in Leonardo the recognition of a higher order, intelligible and perfect. His deep and reflective faith arose less from religious devotion than from a meditation on the order of the world. He sought God in reason and experience, seeing in nature the expression of a creative intelligence.

Leonardo da Vinci also perceived the cyclicity of natural phenomena and the unity of the laws that govern the heavens and the earth. A rigorous observer, he understood that celestial movements are reflected in terrestrial transformations, tides, seasons, the growth of living beings. In the Codex Leicester (folio 9r) he shared his observations:

«L'acqua ritorna per i medesimi luoghi, crescendo e diminuendo per i corsi del sole e della luna.»

"Water returns by the same places, growing and diminishing according to the courses of the sun and the moon."

This observation expresses a cyclical and mechanical vision of nature in which the stars act upon the elements according to physical and measurable laws. In this, Leonardo joined the spirit of the ancient Egyptians who observed the periodic return of the star Sothis (Sirius) as the sign of the renewal of the seasons and of the cycle of life[7].

However, Leonardo distinguished himself profoundly from the superstitious beliefs of his time. He condemned the court astrologers, diviners, chiromancers and other readers of omens, whom he regarded as impostors falsely claiming science. He formulated this idea in the Codex Arundel, folio 176r, British Library:

«Gli astrologi fanno professione di predire i fatti futuri degli uomini, dicendo che essi nascono dalle stelle; ma io dico che, se così fosse, non sarebbe in noi libero arbitrio, e per conseguenza non ci sarebbe scienza né ragione. »

«Astrologers claim to predict the future actions of men, saying that they arise from the stars, but I say that if that were so there would be no free will in us, and consequently neither science nor reason. »

The study of reality thus became for Leonardo a spiritual act. He was convinced that reason and art could raise man toward God. Thus, he made knowledge and creation the paths of an ascent toward the divine. He expressed this conviction in the Codex Arundel, folio 155r, British Library:

«Quella scienzia è più utile della quale il frutto è più simile a quello di Dio; e così la pittura è di tal genere, perché essa è cosa mentale, e per conseguenza partorita da Dio. »

«This science (painting) is the most useful, whose fruit most resembles that of God, and thus painting is of that kind, for it is a thing of the mind and consequently engendered by God. »

2.4. *The Vitruvian Man, a purely geometric construction*

Even before conceiving his Vitruvian Man, Leonardo da Vinci had undertaken an immense preparatory work. His notebooks reveal an accumulation of measurements, dissections and observations. He scrutinized the human being in the smallest anatomical details, calculated the ratios between segments and compared theory with the reality of the body. His drawings bear witness to a quest for harmony that governs the proportions of the human body.

Following the principles set out by Vitruvius, Leonardo first copied the statement of the architect's text, in which the measurements of each part of the body were described. He then added his own observations and notes, comparing the theoretical values proposed by Vitruvius with those he measured on the human body.

To establish his figure, Leonardo drew several horizontal segments intended to mark the principal divisions of the body, one at the level of the neck, another at the height of the thorax, another at the level of the pubis and a final one at the height of the knees. These reference lines, visible on the drawing, served to measure the length of the different parts of the body and to verify their proportional ratios.

His notebooks dated from 1489 to 1498 bear witness to a reflection between anatomy, the mechanics of movement and the mathematical principles of equilibrium. He observed (Anatomical Manuscript A, f. 2r, Royal Library, Windsor Castle):

«Il movimento delle membra è governato dalle proporzioni.»

« The movements of the body are governed by proportions. »

The geometric analysis of the drawing reveals an internal organization of great precision (figure 30). The choice of fractions used by Leonardo, expressed on a base of 120 divisions, makes it possible to obtain precise and harmonious ratios in the structure of the body and of space.

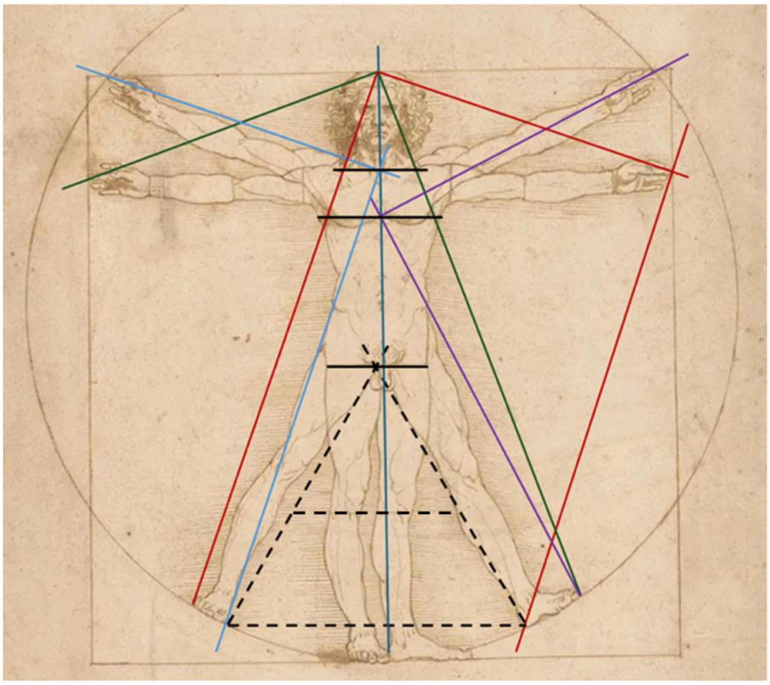


Figure 30. *Geometric construction of the Vitruvian Man, Da Vinci Gallerie dell'accademia of Venise*

The positioning of the principal joints of the Vitruvian Man, combined with the segments drawn by Leonardo on the body, constitutes the anchoring points of a geometric system. This arrangement obeys a law of projection which generates right angles converging toward significant points of the drawing.

Indeed, if one draws a segment from the right hand placed in three-quarter view and situated precisely on one of the points of intersection between the circle and the square, to the center of the body at the level of the neck, it forms a right angle with the segment that runs from the middle of the neck to the inner extremity of the right foot in extension (see blue segments).

Likewise, the segment that links the left hand in three-quarter view to the middle of the segment that marks the bust forms a right angle with the segment that links the middle of the bust to the outer extremity of the left foot in extension (see violet segments).

Moreover, the segment that runs from the middle of the top of the head to the extremity of the horizontal left hand forms a right angle with the segment that runs from this latter point to the inner extremity of the left foot in extension. Another right angle is formed by drawing a segment from the top of the head to the outer extremity of the right foot in extension (see red segments).

In a symmetrical way, the one that runs from the top of the head to the extremity of the horizontal right hand forms a right angle with the segment from the top of the head to the outer extremity of the right foot in three-quarter view (see green segments). Finally, the segments that link the top of the head to the outer extremities of the extended feet, when aligned with the central axis of the body, form two equal right angles which ensure the equilibrium and symmetry of the figure.

Among the instructions stated by Vitruvius appears the following:

"If you spread your legs so as to diminish your height by one fourteenth and extend and raise your arms until the tips of your fingers reach the height of the top of the head, know that the center of the extended limbs is located at the navel and that the space between the legs forms an equilateral triangle."

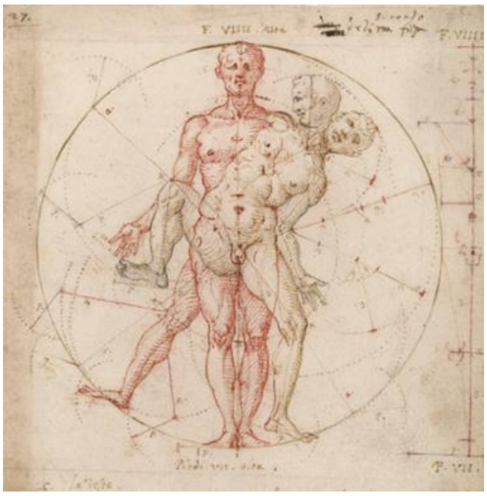
Yet in his drawing Leonardo did not strictly follow this indication. He corrected it. After having studied the anatomy of man he observed that the navel is not situated at the middle of the body. He displaced the center to the level of the pubis. This slight shift was indispensable to the geometric coherence and anatomical fidelity of the figure.

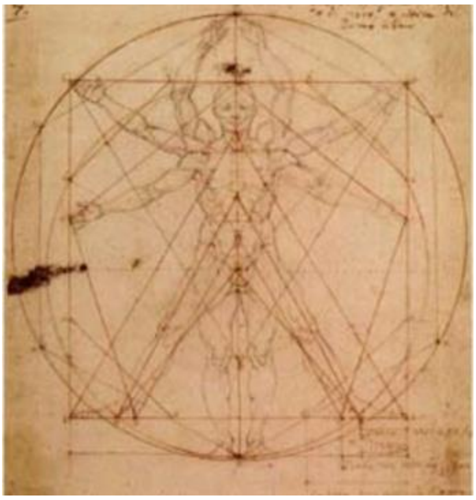
It is from this center that the equilateral triangle mentioned by Vitruvius is naturally formed. Its base corresponds to the inner extremities of the extended feet and its apex is situated exactly at the center of the pubis. The triangle is not centered at the middle of the body because the legs are slightly in movement. This subtle displacement bears witness to the dynamics of living form. Finally, if one prolongs the segment drawn by Leonardo at the level of the knees another equilateral triangle is formed inside the first.

This subtle equilibrium between geometric rigor and anatomical observation manifests the full complexity of the approach. Leonardo knew the human body intimately, having observed and studied it in the smallest detail. He understood gravity, the tension of the muscles, the resistance of the bones and the balance of the masses. Each position, each inclination of the torso, each support of the foot corresponded to a measured reaction throughout the body. If the arm was raised the weight shifted, the axis corrected, the hip compensated. This awareness of movement and forces gives his drawing a real stability, almost living.

In this constant correspondence between the movement of the body and measurement emerges the invisible armature of a mathematical order. All these ratios reveal an underlying geometric grid governed by an internal harmony that organizes the body into a proportional and balanced whole. Such a conception bears witness to a rare mathematical ingenuity. Leonardo succeeded in making the rigor of calculation coincide with the complexity of the living body. He simultaneously resolved a set of metrics, angular and proportional constraints while unifying in a single structure the requirements of measurement and those of nature. The representation of the body of the Vitruvian Man thus became a true geometric demonstration.

Through this scientific approach Leonardo da Vinci solved the problem posed by Vitruvius, but without ever revealing the solution. His followers, such as Bernardino Luini, copied or reused his notebooks without grasping the geometric logic of the drawing. The Codex Huygens[57], produced around 1570 by the Milanese painter Carlo Urbino, attests to this. This manuscript gathers numerous drawings that reprise and comment on Leonardo's studies, including the Vitruvian Man itself (figures 31). But far from rediscovering the clarity of the master these copies bear witness above all to a laborious and unfinished search, a sign that his imitators failed to grasp fully the solution that Leonardo had given.





Figures 31. *Codex Huygens, Carlo Urbino, Fol.6 et 7, 1560/1570*

3. Leonardo's da Vinci Sculptural Project of the Ideal Horse

3.1. *Historical Context*

Leonardo da Vinci conceived his Vitruvian Man in Milan, while he was in the service of Duke Ludovico Sforza. It was in this intellectual and artistic environment of exceptional richness that he undertook his research on the proportions of the human body. It was precisely within this period of intense scientific and artistic activity that Leonardo met the mathematician and Franciscan friar Luca Pacioli. Around 1496 the latter was summoned to Milan by Ludovico Sforza[58]. There he met Leonardo da Vinci, already active at the court for more than a decade. The two men formed a friendship and collaborated closely between 1496 and 1499. Contemporary sources report that they resided in the same house[59], situated near Santa Maria delle Grazie, the very place where Leonardo painted The Last Supper. This proximity favored sustained intellectual collaboration centered on the study of proportional ratios, perspective and geometry.

In 1498 Pacioli composed his great treatise De Divina Proportione, devoted to the divine proportion, to the golden ratio. He stated:

«Questa proporzione noi chiamiamo divina, perché simile a Dio ;e nelle opere di Dio si trova, e principalmente nel corpo dell'uomo,che si può dire modello e misura d'ogni cosa creata. »

« We call this proportion divine, because it resembles God; one finds it in all the works of God, and principally in the body of man, which may be considered the model and measure of all created things. »

Leonardo personally illustrated the treatise by producing the regular and semi-regular polyhedra, drawn with remarkable geometric rigor and intended visually to demonstrate the construction of the golden ratio[29]. In his preface Pacioli explicitly thanked him:

«A Leonardo da Vinci, excellentissimo pittore et perspectivo, devo le mirabili figure che adornano questo libro. »

« To Leonardo da Vinci, most excellent painter and perspectivist, I owe the marvelous figures that adorn this work. »

This collaboration deeply marked both men. Pacioli regarded Leonardo as "il più gran maestro della geometria applicata all'arte[59], the greatest master of geometry applied to art. It is likely that the choice of Leonardo to illustrate his work was not accidental. The Tuscan genius had already been pursuing for several years a parallel reflection on the proportion of the human body, of which the Vitruvian Man constitutes the synthesis. This celebrated drawing, produced around 1490–1492, formed part of an approach in which the human figure became the very model of measurement and universal harmony. Leonardo and Pacioli thus shared this common vision of a rational order inherent in every form of perfection.

This affinity between Leonardo and Pacioli also took root in the Franciscan intellectual tradition. Leonardo himself alluded in the Codex Arundel to two major figures of this tradition. In Codex Arundel, 71v he mentioned: *"Rugieri Bacon fatto in istampa"*, which corresponds to Roger Bacon[60], philosopher, scholar and English alchemist of the thirteenth century, regarded as one of the precursors of the experimental method. He also cited *"Cerca in Firenze della Ramondina"* in Codex Arundel, 192v, which refers to Ramon Llull, Catalan theologian and missionary. Both belonged to the Franciscan order and sought to unite faith, reason and science in the understanding of the world. Bacon advocated knowledge founded on observation and experience, while Llull conceived in his Ars magna[61] a geometrical and spiritual logic intended to reveal the divine order of creation.

These references bear witness to the interest Leonardo showed for this mode of thought in which measurement and geometry are perceived as paths toward truth. In this context his closeness with the Franciscan friar Luca Pacioli and his insertion in the religious circles of Milan, as is shown notably by the commission of the Virgin of the Rocks[62] for the Franciscan confraternity of the Immaculate Conception, link him fully to this intellectual tradition.

Moreover, Leonardo shared with the disciples of Saint Francis a taste for simplicity and detachment from wealth. He often mocked the vanity of luxury and the futility of power, opposing to them sobriety, contemplation and the work of the mind. For him (Codex Arundel, folio 155 recto, British Library):

«Il più nobile diletto è la gioia dell'intendere.»

« The noblest pleasure is the joy of understanding. »

In the Codex Trivulzianus, f. 86r, he shared a note on the vanity of man:

«O vana speranza, quanti ti seguono e quanti ti perdono!»

« O vain hope, how many follow you and how many are lost in your wake! »

And elsewhere on the same folio:

«O misera umanità, di quante cose ti rendi schiava per vile guadagno!»

« O miserable humanity, how many things enslave you for a paltry gain! »

At this same period Leonardo was working on his project of the ideal horse intended to honor the memory of the father of Duke Ludovico Sforza[63,64], a project to which he applied the same principles of equilibrium and proportion as those developed in the conception of the Vitruvian Man. He declared in the Codex Madrid II, 2r:

«Farò un cavallo di bronzo, grande, che non fu mai fatto di simile grandezza, e di proporzioni perfette.»

« I shall make a horse of bronze, large, such as never has been made of similar size, and of perfect proportions. »

3.2. *The Pursuit of Perfection trough Drawing*

Commissioned in the early 1480s this first equestrian project occupied Leonardo for more than a decade. After many years of studies, he produced a plaster model presented in 1494 on the marriage of Bianca Maria Sforza with Maximilian of the Holy Empire. At the sight of this model the poet Piattino Piatti, friend of the duke, composed a poem in homage to this work[65], he compared Leonardo to the great sculptors and painters of antiquity, emphasizing how much this model represented a search for equilibrium and perfect proportions worthy of ancient ideals.

To succeed in conceiving a horse with the aim of surpassing the great models that preceded it, from Donatello's Gattamelata to Verrocchio's Colleone, Leonardo filled his notebooks with observations, sketches and calculations which constituted true research laboratory[66]. Each drawing aimed to define with extreme precision an anatomical part of the animal. Leonardo's ambition was to combine the most remarkable elements observed in different horses[67], to create a horse that was both real and imaginary.

The abundance and diversity of his studies bear witness to the magnitude of this graphic enterprise. Leonardo multiplied views from every angle and experimented with various materials, charcoal, black chalk, red chalk, to explore volumes, muscular dynamics and the effects of light. This research resulted in an exceptional drawing in red chalk identified as the most accomplished version of the ideal horse[68] (figure 32).

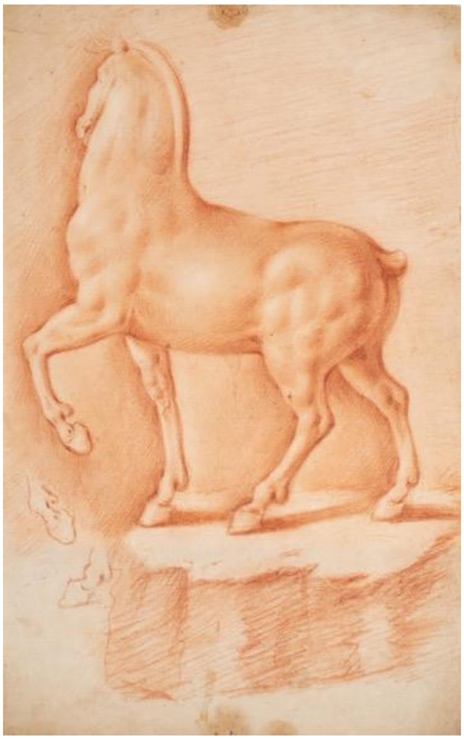


Figure 32. *Ideal Horse, recto, 45,3 cm x 27,5 cm*
Red Chalk drawing, Private Collection
Da Vinci

On the reverse of this drawing is another study depicting a horse executed in a freer and more spontaneous manner (figure 33). It is a sketch probably made from direct observation in the stables, as in several of Leonardo's studies preserved today at the Royal Collection Trust in Windsor (figures 34). Leonardo frequently reused his sheets, drawing on both the recto and the verso, which attests to a constant practice of observation[69]. He also had the habit of cutting his sheets. One can see that part of the horse's head is missing, as in the drawings below from the Royal Collection Trust.

Figure 33. *Verso Ideal Horse*

Figures 34. *Studies of horses, Codex Windsor, Rcin 912309, Rcin 912312*

The technique used in the drawing of the Ideal Horse is strictly identical to that observed in several studies by Leonardo preserved at the Royal Collection Trust. In the study of the child (figure 35), that of the dog (figure 36), the nude man seen from the back (figure 37), and in a study of a horse (figure 38), Leonardo used red chalk, a medium he mastered with exceptional precision, to model forms with suppleness and to give them a striking physical presence. The contour, laid down with a continuous and fluid gesture, varies according to the light, pressed more firmly in shadow and lightened in illuminated areas. It is through this modulation of the line that a play of shadow and light is established, where volume emerges from the very breathing of the stroke. This contour defines the optical and anatomical structure of the subject, the visual framework of the body and its orientation in space.

Figure 35. *Study of a child*

Figure 36. *Study of a dog*

Figure 37. *Nude man from the back*

Figure 38. *Study of a horse*

In the drawings above, as in the study of the Ideal Horse, the modelling is then constructed through a network of oblique and curved hatching applied in several successive passes (figure 39). Their orientation depends on anatomical relief, the movement of the body, and the direction of the light. These hatchings, which reveal the internal dynamism of the model, express the tension of the muscles, the weight of the masses, and the continuity of the planes. The bundles tighten in areas of pressure and open in areas of release, according to the functional logic of the body. In these studies, Leonardo adjusted the pressure of the stroke denser over bony protrusions, lighter over soft flesh. He also occasionally softened transitions with a slight blending, applied with the finger or a stump, without erasing the tooth of the paper. This method creates a diffuse yet precise modelling based on the variation in pigment density.

Figures 39. *Details of modeling and hatching networks in Leonardo da Vinci's red chalk studies*

In the study of the ideal horse, the reserved highlights are distributed with remarkable exactness (figures 32, 39). They correspond to the areas where light glides across the relief, the withers, the croup, the chest, the spine, and contribute to the overall balance of the form. Their role is to allow the material to breathe. Leonardo exploited the tone of the support as a luminous source, he played with the tint and texture of the paper to create an internal light that seems to emanate from the body itself. This technique gives the drawing a vibrant presence, where light becomes a means of construction as much as perception. It is one of the most characteristic principles of Leonardo da Vinci, which no disciple ever equaled.

The Horse's Head (figure 38) preserved at the Royal Collection Trust (Codex Windsor) fits exactly within this method. Attributed to Leonardo da Vinci since always, it comes from the collection of Francesco Melzi, direct heir to the master's drawings, then passed to the sculptor Pompeo Leoni, then to Thomas Howard, Earl of Arundel, before being acquired by King Charles II, around 1690. This chain of provenance, perfectly documented, links the sheet to the Leonardian corpus. It was only in the twentieth century, in the 1930s, that the historian Kenneth Clark proposed a reattribution to Cesare da Sesto, based on the rightward inclination of the hatching. Such a hypothesis, purely formal, belongs to a critical trend of the time that sought to reduce the autograph corpus of Leonardo by attributing a significant part of his studies to his students. Numerous sheets manifestly autograph were thus unjustly excluded, which weakened the understanding of Leonardo's graphic work[70]. Yet, when replaced in the context of Leonardo's activity, the Horse's Head (figure 38) naturally finds its place.

Leonardo da Vinci devoted nearly fifteen years to the study of the horse, from the research for the Sforza monument to the projects for the Trivulzio monument, multiplying anatomical and mechanical analyses of movement. He even undertook the writing of a complete treatise dedicated to the animal, intended to accompany the monumental horse project. This work, mentioned by his contemporaries and evoked in his notes, was unfortunately lost[66]. It survives only through a few scattered leaves and preparatory studies preserved at Windsor.

After the death of Leonardo da Vinci in 1519, his pupil Francesco Melzi took with him to Lombardy most of the master's notebooks and drawings[71]. These thousands of folios contained studies of anatomy, mechanics, geometry and proportions, which were jealously kept in Melzi's villa at Vaprio d'Adda until his death, around 1570, then gradually dispersed by his descendants, who understood neither their value nor their scientific significance.

A large portion passed into the hands of the Milanese sculptor Pompeo Leoni[72], in the service of the king of Spain Philip II, who reorganized the whole, arranging, cutting and mounting the pages to form vast volumes, today known as the Codex Atlanticus, Codex Madrid and Codex Windsor. This operation also caused the physical and thematic dispersal of the sheets, many of which were sold or given to Italian and foreign collectors.

Carlo Pedretti notes that *« certain studies of horses in red chalk, mentioned in ancient inventories, must have been detached from Leoni's volumes and are today lost or unlocated[73].* »These lost studies appear in the inventories drafted after Leoni's death, preserved in the collections of the Biblioteca Ambrosiana in Milan and the palace of El Escorial[74]. Several entries mention "drawings of horses" and "studies of limbs in motion," which likely circulated independently of the volumes before passing into private Italian collections, then to England[69].

In the seventeenth century, the great English collector Thomas Howard, 14th Earl of Arundel, assembled in London one of the most important art collections in Europe[75]. It included notably numerous drawings by Leonardo da Vinci. The diarist John Evelyn[76] noted in his journals: *« drawings and manuscripts by Leonardo da Vinci filled with studies of movement, anatomy and mechanics. »* In a contemporary letter, William Petty also mentioned[77]: *« drawings in red chalk representing the parts of the body and the mechanisms of movement. »*

After the death of the Earl of Arundel in 1646, his heirs began selling large portions of the collection, some manuscripts joining the Royal Collection, such as the Codex Arundel, now at the British Library, and several drawings preserved at Windsor, while other lots went onto the London art market[78]. Evelyn noted the visit of "many visitors from the former Low Countries," who came to view or purchase Italian drawings. In the following century, London auction catalogues continued to record Italian sheets "from the collection of the late Earl of Arundel," acquired by dealers from Amsterdam and Antwerp. Historians Francis Haskell and Nicholas Penny summarized this circulation[79]: "Part of the drawings from the Arundel collection passed into the hands of Dutch and Flemish dealers active in London, including the Uylenburgh family[80], who traded between Amsterdam, Antwerp and Brussels."

The specialist Carlo Pedretti, who devoted a large part of his life to the study of Leonardo's manuscripts, recalled that this dispersion "shattered the coherence of Leonardo's graphic work, so that what belonged to a single treatise ended up separated across countries and centuries." He also stressed that the absence of provenance should not cast doubt on the authenticity of a sheet, for many autograph drawings known today, notably in Turin, Florence or Windsor, resurfaced after several centuries of oblivion and may still reappear in private collections[69].

The examination of the current Leonardian corpus reveals indeed an abundance of sketches and analytical studies on the theme of the horse, often drawn with pen or black chalk, intended to explore proportions, movement mechanics or muscular structure. By contrast, fully constructed studies in red chalk are exceptionally rare. Yet this stage aligns logically with the continuity of his practice. Leonardo experimented successively with each medium to exploit its properties[81]. Red chalk, by its suppleness and its ability to unify line and light, allowed him to approach the question of modelling with a precision that neither pen nor black chalk could offer.

This experimental approach is manifest in a drawing preserved at the Royal Collection Trust, which represents the body of a horse in profile, executed in metalpoint on blue-prepared paper with white heightening (figure 40). In this study, Leonardo attempted to render volume through the sole contrast of metal and heightening, but the result remains limited, as this medium did not allow him to obtain the density nor the fluidity of modelling he sought. The transition toward red chalk thus appears as a necessary step in the natural progression of his research, as demonstrated in his studies of the human body, where each medium serves a specific analytical purpose. He stated in his Treatise on Painting[31]:

«I pittori debbono avere conoscenza dei colori e de' loro misti, e de' varii modi del disegno, sì col carbon nero come col gesso o pietra rossa.»

« Painters must know the colors and their mixtures, as well as the various means of drawing, in black charcoal, in chalk, or in red stone. »

Figure 40. *Étude du corps du cheval de profil, Rcin 912289 da Vinci,Royal collection Trust*

The extreme attention that Leonardo devoted to the representation of the horse, and the long series of studies he dedicated to it, may be explained not only by his scientific interest and his genuine attachment to the animal, but also by the major role the horse occupied in the culture of his time.

At the Renaissance, the horse played a major role in visual and political culture[67]. In the great equestrian commissions, such as those of Ludovico il Moro, the horse embodied military power, nobility and the rank of its rider. The animal was conceived as an extension of the prince, its strength and bearing reflecting those of its master.

Leonardo da Vinci's horses are imposing, with powerful necks, a frequent detail is the cut or shortened tail. Caudectomy was a common practice in the Renaissance to prevent the tail from hindering the rider during combat or hunting (figures 41).

Ideal Horse, Private Collection

Studies of horses, Royal Library

The imposing neck of Leonardo da Vinci's horses

Studies of horses Museum of Louvre

Tail docking

Study of horse British Library

Figures 41

The drawing of the ideal horse, which may be considered as a true pre-sculptural study, corresponds to an advanced stage. In this respect, it provides valuable information on the way Leonardo conceived sculpture, its monumentality and especially his spatial conception. Leonardo implemented a perfectly mastered perspective through a precise internal organization that gives the composition real three-dimensional depth. The sketched base on which the horse stands is slightly inclined and seen from above, which places the viewer below and accentuates the sensation of height (figure 42). This device enables the equestrian figure to dominate visually, as it would in public space upon its pedestal.

This base was sketched using zigzag hatching, like that of the portrait of Cesare Borgia (figure 43). This rapid technique allowed him to cover areas or indicate shadows while sometimes concealing earlier sketches, the sheets being frequently reused.

Figures 42. *Ideal Horse*
Private Collection
Da Vinci

Figure 43. *Portrait of Cesare Borgia*
Inv.15573, Royal Library of Turin
Da Vinci

3.3. *The Geometry of living forms, from man to animal*

Leonardo inherited a visual context shaped by the tradition of the monumental horse, but he developed a personal vision founded on a far more advanced anatomical approach, derived from direct observation and scientific study. He transposed into it the same principles that he applied to the human body.

Like for the Vitruvian Man, Leonardo began by constructing a precise geometric structure, in which the body of the animal is inscribed within a network where each part is determined according to rigorous proportional ratios. This method appears in a drawing devoted to the study of a horse, entirely governed by internal geometric relationships (figure 44). Each segment corresponds to precise mathematical proportions, demonstrating that the figure results from a rational construction prior to any artistic intervention.

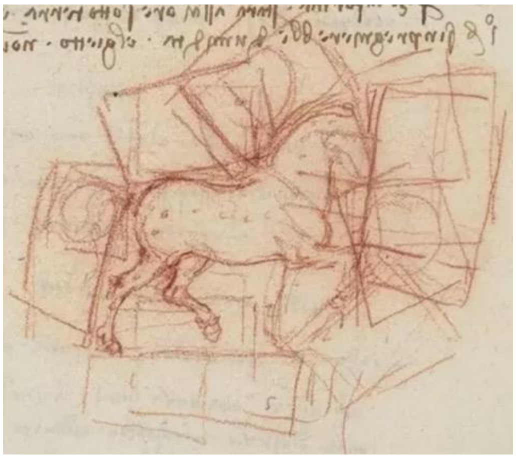


Figure 44. *Study of horse, Codex Madrid II, folio 151v.*

The hindquarters of Leonardo’s horses are organized according to a circular structure whose center corresponds to the hip joint (figure 45). This geometric device reflects the real functioning of the hind limb, propulsion resulting from a rotational movement around this point. The circular line that links the hip, the thigh and the hock reveals the internal dynamics of movement.

Ideal Horse, Private Collection

Studies of horses, Royal Library of Turin

Figures 4. *Da Vinci*

Leonardo’s objective was to reveal the existence of a universal harmony shared by all living forms, whether the human body, the animal world, music, or nature itself. He applied the same geometric principles and the same laws of proportion to the horse as to the Vitruvian Man. The superimposition of the two figures (figure 46), although their absolute measurements differ, reveals that they share the same proportionality. The framework of lines drawn by Leonardo on the Vitruvian Man serves as a rule for both bodies. Each segment meets an anatomical reference point of the horse, as though a single mathematical grid ordered the anatomy of both beings. This correspondence recalls the principle stated by Vitruvius according to which « *man is the measure of all things*[40]. » The human body here becomes the standard by which Leonardo evaluates and harmonizes the body of the horse.

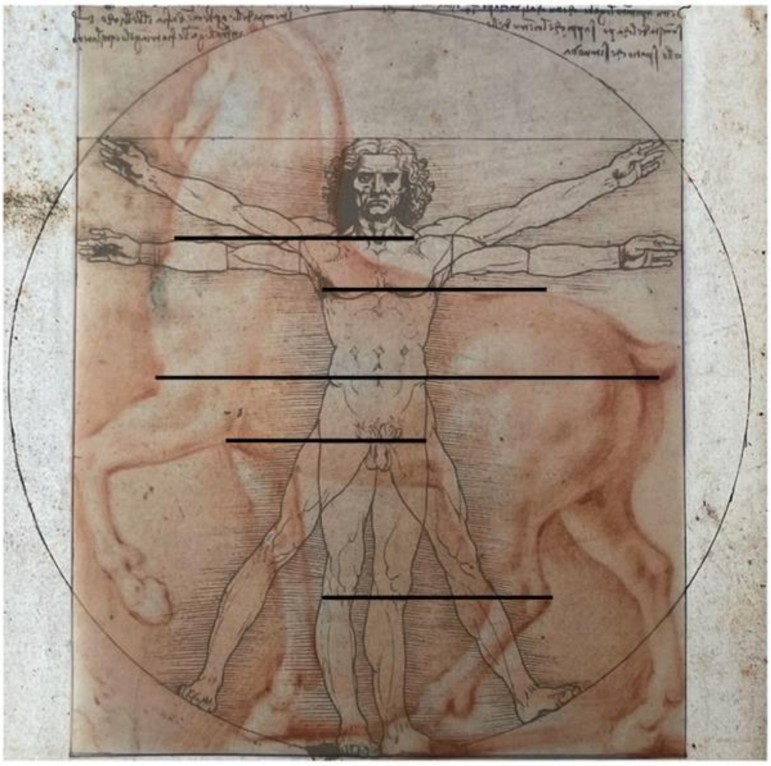


Figure 46. *Transparent juxtaposition of the Ideal Horse and the Vitruvian Man*

Indeed, the height of the base of the man’s neck coincides with the withers of the horse. The line of the torso meets the croup, the line of the pubis corresponds to the junction of the two forelimbs, and the line of the knees falls at the level of the carpus of the flexed foreleg. The center of the horse, that is, half of its height, is located at the level of the man’s navel. It also corresponds to the level of the animal’s pubis, that is, the midpoint of its body just as in the Vitruvian Man (figures 46,47).

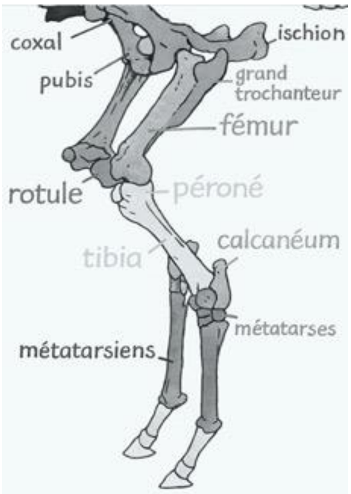
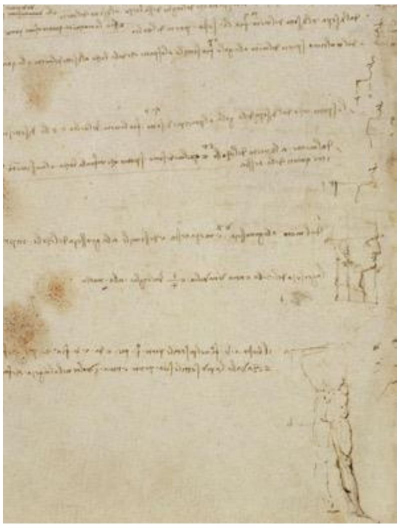


Figure 47. *Drawing of the anatomy of the horse, position of the pubis*

Many studies by Leonardo, on human and equine proportions, are found on the same sheets, which demonstrates a comparative approach (figures 7, 48). Taking up the idea expressed by Aristotle[34], according to which the legs of man and those of the horse share an analogous structure, the knee of the man’s right leg, set apart, meets that of the horse’s right foreleg, while the left joined foot of the man aligns with the flexed left hind hoof of the animal.



Verso

Recto

Figures 48. *RCIN 912304, Windsor collection, Da Vinci*
On the same sheet, studies of equine and human proportions

Thus, through this anthropometric conception, derived from the Vitruvian model, Leonardo illustrates the idea that the internal ratios of the human body embody the rational order of the world and may be applied to any construction, whether architectural, mechanical or artistic.

This drawing also bears the imprint of Leonardo’s anatomical rigor. The horse is represented in motion, one foreleg raised and the other perfectly extended, while the head turns upward to the right. Indeed, in the horse, the cervical vertebrae do not allow a purely vertical elevation of the skull. To raise its head, the horse must combine an extension and a lateral rotation of the neck. This movement produces a tension throughout the body, which causes the medial saphenous vein to appear prominently along the extended leg, visible beneath the skin (figure 49). This anatomical detail, rendered with remarkable accuracy, attests to Leonardo's scientific precision. The muscular tension and the ascending line formed by the neck and the head define the maximal measurable height of the horse, without breaking the balance of the movement. Leonardo thus conveys, with the precision of an anatomist, the tensions and internal structures of the animal.

However, this horse remains an idealized creation. Leonardo, while relying on rigorous observation of reality, adjusts certain details to make it a sculptural work. This is why the curvature of the hoof is raised to ensure the stability of the whole, in accordance with an equestrian tradition inherited from Antiquity and taken up by the masters of the Renaissance.

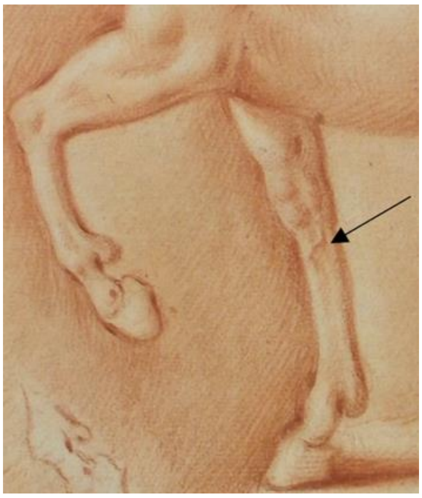


Figure 49. *Ideal Horse, medial saphenous vein*

3.4. *The mastery of matter and movement*

In the conception of this large-scale sculptural project, Leonardo combined the science of proportions with a true reflection on engineering, where the mechanics of the animal body become the model of a living, stable and rational architecture. Conceived according to a rigorous geometrical and mechanical planning, this undertaking aimed at structural solidity, the control of masses and the overall balance of the composition. Leonardo thus organized the whole around an orthogonal grid, which divides the surface into proportional modules, and defines reference axes, which serve for the placement of volumes and the orientation of internal forces.

In his studies on equine proportions (figure 50), Leonardo established that the span of the horse's head and neck represents about 30% of its body. This proportional distribution, which appears in the study of the ideal horse, forms the basis for the conception of the body, like a balanced force system governed by the laws of static mechanics. Thus, the croup and the hindquarters, which ensure resistance, occupy 60%. The remaining 10% constitute a zone of mechanical transition, which absorbs opposing forces, connects the masses and ensures the continuity of constraints between the traction of the front and the push of the rear. The withers act as a pivot.

This ratio of 1/2 between the front and rear masses corresponds to a mechanical principle observed in systems of leverage and weight distribution. A small force applied to a short arm can balance a larger mass on an arm twice as long, thereby ensuring a stable equilibrium. On the ideal horse, the tension of the neck (traction), which opposes the push of the hindquarters (compression), generates a stable mechanical moment and continuous dynamic compensation.

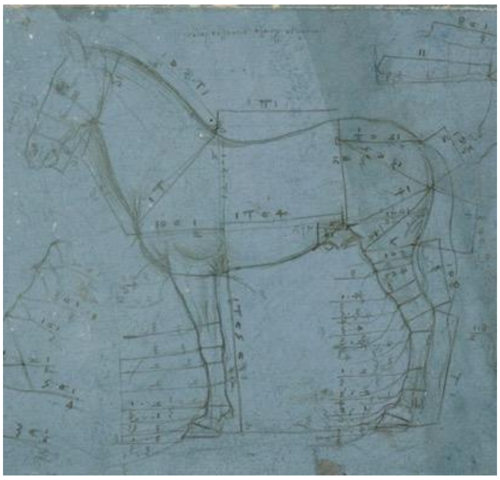


Figure 50. *Study horse's proportion,*
Rcin 912319, Codex Windsor
Da Vinci

From this rational organization arises, in the study of the ideal horse, a true geometric framework, composed of two equal right triangles, which structure the composition (figure 51). The first connects the head to the supporting foreleg, the second the withers to the croup and hindquarters. Their right angles lock the direction of the loads, and their hypotenuses converge toward a single gravitational point, located in the lower part of the central vertical axis. This static node concentrates the vectors of traction, compression and reaction. All the principal lines, the neck, the back, the limbs and the croup, converge there. This point acts as a pivot for the redistribution of mechanical forces toward the supports. Through this convergence, the forces partially cancel each other, those pulling forward and those pushing backward compensate each other. This balance allows the horse to maintain a stable and harmonious posture.

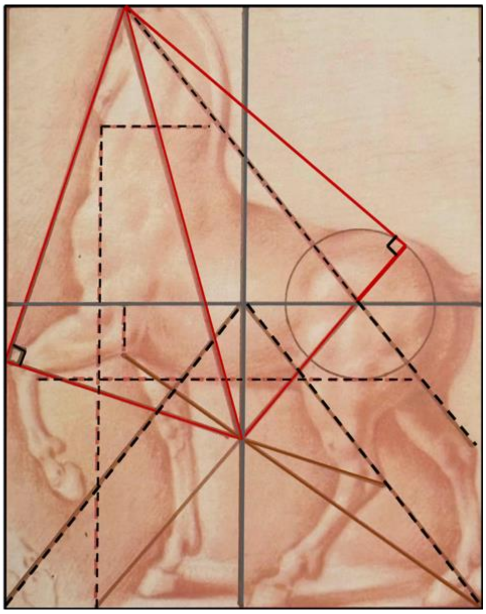


Figure 51. *Geometric framework of the Ideal Horse*

The hindquarters, conceived according to a circular geometry, complete this structure. The croup, which forms an arc of rotation centered on the hip joint, acts as a lever of propulsion. This curvature transmits the linear forces of the back in the form of rotational movement, which ensures the forward thrust. In mechanical terms, this zone functions like a natural pulley, stabilizing the posterior mass while compensating for the loads generated by the tension of the neck. The entire body thus behaves like a living triangulated beam, organized around a gravitational point and supported by a circular base of rotation. This conception reflects Leonardo da Vinci's assimilation of the works of Archimedes[82]. Thus, beyond the equestrian representation, this drawing reveals an engineer's way of thinking fully integrated into artistic creation. Each line responds to a precise structural function, which makes this study a true modelling of statics. It anticipates the technical constraints of the future bronze casting. This conception corresponds to the mechanical principles formulated by Leonardo[83] in the Codex Madrid I, F 10v:

«Ogni peso che ha il suo centro di gravità sopra il suo sostegno non cade, ma quando lo passa cade, e quanto più lo passa tanto più presto cade.»

« Any weight whose center of gravity is above its support does not fall; but once it passes beyond it, it falls, and the more it passes beyond it, the faster it falls. »

4. The Anatomy of Nature

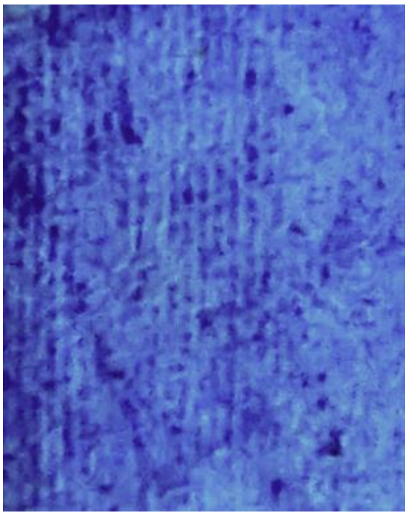
Between 1482 and 1499, Leonardo produced numerous anatomical and mechanical studies intended to design a monumental bronze horse for Duke Francesco Sforza. These investigations, grounded in geometry, mechanics, engineering, anatomy, and the physics of movement, already testified to his desire to unite science and art. After the fall of the Duchy of Milan in 1499, Leonardo continued his studies of the ideal horse, which he refined in the light of his discoveries in the fields of statics and applied mechanics.

The drawing of the ideal horse marks a decisive stage in the evolution of his work. Around 1508[84], Leonardo applied to the Trivulzio horse a more dynamic conception while reintroducing the anatomical, mechanical, and geometric principles developed two decades earlier for the Sforza horse. Left unfinished, this project reflects a more mature approach, extending the research undertaken for the monument of the Duke of Milan.

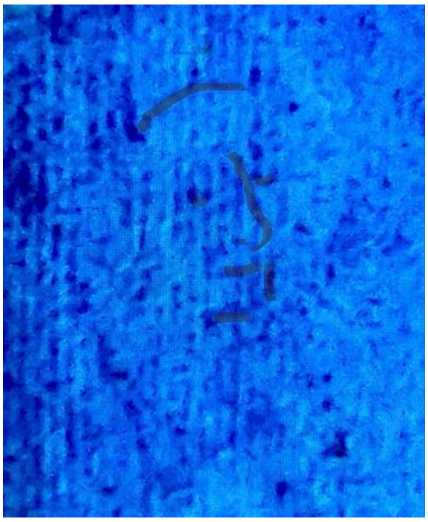
To devote himself fully to this new sculptural project, Leonardo interrupted the execution of a religious work (the Virgin, Saint Anne, the Christ Child and Saint John the Baptist), begun a few years earlier, around 1501–1503[85], in Florence. But upon the death of Marshal Trivulzio and in the face of political instability in Milan, the undertaking was abandoned. Leonardo then returned to his research on the Virgin and Saint Anne, around 1510/1513.

Scientific ultraviolet imaging carried out on the reverse of the ideal horse confirms this period of artistic transition. A sketch of the Virgin, corresponding to the drawing preserved in the Louvre (figures 54, 55), appears. If it becomes visible under ultraviolet but remains invisible in infrared, this is due to the presence of iron oxide in the pigment. The analyses conducted by the Université Pierre et Marie Curie (UPMC/CNRS, ISTEP) confirmed a high concentration of iron oxide in the material, identified as red hematite, frequently used by Leonardo. This compound strongly absorbs infrared radiation, while reacting under ultraviolet light with a faint reddish fluorescence, which allows the drawing to become perceptible[86]. This phenomenon was noted by the Italian scientist and conservator Antonino Consentino[87]. It is more remarkable that the final panel preserved in the Louvre (figure 56) reveals, under infrared imaging, a horse's head (figure 57), which demonstrates the transition from the conception of the Trivulzio horse to the elaboration of the religious painting.

These material and scientific indications help reposition, with new precision, the different stages of Leonardo's work and the chronology of his research. They testify to the contemporaneity of his studies, carried out simultaneously in fields as diverse as sculpture, painting, mechanics, and cartography.



Underlying drawing of the Virgin
UV reflectography on the reverse of the Ideal Horse
Da Vinci



Revealing the underlying drawing

Figure 54

Figure 55. *Preparatory drawings « The Virgin, Saint Anne and the Christ Child »*
Da Vinci

Figure 56. *The Virgin, Saint Anne and the Christ child*
Da Vinci

Figure 57. *Revealing the underlying drawing*
Da Vinci

At the beginning of the sixteenth century, after having returned to Florence following his service with Cesare Borgia, Duke of Romagna, Leonardo placed his skills as an engineer and cartographer at the service of the Florentine Republic, which entrusted him with the study of the course of the Arno and its possible diversions. This mission coincided with the first phase of work on The Virgin and Saint Anne.

In Florence, Leonardo found a city engaged in large-scale hydraulic engineering projects. Under the gonfalonier Piero Soderini and the secretary Niccolò Machiavelli, the Florentine Republic was considering diverting the course of the Arno to link the city to the sea and weaken the power of Pisa. As early as the summer of 1503, a letter from Machiavelli explicitly mentioned his mission to study the river and its possible diversions[88].

Between 1503 and 1504, he produced several maps of Tuscany and of the Arno valley, today preserved at Windsor (figures 58). These maps, of large dimensions, were executed in pen and ink, and heightened with colored washes. Their support is a heavy-weight paper, thick and resistant, corresponding to high-grade administrative papers, very different from those used in artists' workshops.

Carte de la Vallée d'Arno, 27,5cm x 40,1cm
Rcin 912683, Royal Collection Trust
Da Vinci

Carte rivières et montagnes, centre d'Italie,
31,7cm x 44,9cm
Rcin 912277, Royal Collection Trust, Da Vinci

Figures 58

The analysis of the paper of the study of the Ideal Horse, carried out by Stefano Fortunati, specialist in ancient documents, in collaboration with Silvio Balloni, curator of the Ginori Lisci archives in Florence, revealed that it was an administrative paper of the same type as that used in chanceries, registers, accounting books and notarial studies for the drafting of official acts. Moreover, the format of the sheet (45.3 × 27.5 cm) corresponds closely to that of the large topographical maps at Windsor (figures 58).

The use of these large notarial sheets, capable of absorbing-colored washes and supporting reworking with the pen, suited perfectly the technical requirements of cartography as well as of his anatomical research and preparatory drawings in red chalk. Its slightly grainy surface and density allowed for a fine adhesion of pigment, favoring the modelling effects and subtle gradations he sought in his studies of volumes and drapery. This robust paper, designed to last, thus offered Leonardo an ideal support for both his scientific and graphic work.

This type of paper, several sheets of which used by Leonardo bear watermarks attributable to the mills of Fabriano and more rarely of Foligno[89], corresponds exactly to the high-quality production that circulated in Tuscany.

Furthermore, the fact that this paper is of the same type as that used in registers and official acts gives resonance to the presence in Florence of Leonardo's father, Ser Piero da Vinci, notary of the Republic, who died in 1504, when Leonardo was working on his maps and his major pictorial projects[90].

The cartographies of Leonardo present a topographical accuracy verified by modern geographers. Distances and orientations are represented with astonishing precision for the period, obtained by hand and without any modern measuring instruments.

Leonardo succeeded in rendering the dynamics of relief, watercourses and winds, transforming the map into a true living anatomy of the territory. He conceived the earth as an organism in motion, governed by internal forces analogous to those of the human body. In his maps, rivers are the veins of the earth, valleys its muscles and mountains its skeletal structure, while water plays the role of blood, the vital fluid that irrigates and shapes the landscape. He formulated this idea, around 1504, in the Codex Leicester, F.10.r.:

«Il sangue che scorre per le vene dell'uomo è simile all'acqua che scorre per le vene della terra; come il sangue nutrisce e vivifica il corpo umano, così l'acqua nutrisce e vivifica la terra.»

« The blood that flows through the veins of man is like the water that flows through the veins of the earth; as blood nourishes and gives life to the human body, so water nourishes and gives life to the earth. »

This analogy reveals a profoundly unified conception. Through his representations of the territory, Leonardo sought to reveal the physiology of nature, a system of balances and circulations in which each element is bound to the other.

This idea is attested in the Codex Arundel, f.155 r.:

«Tutte le cose sono legate insieme da vincoli che l'occhio non può vedere.»

« All things are bound together by links that the eye cannot see. »

Thus, where Mesopotamia inaugurated the idea of measurement and the existence of a rational order in nature, Leonardo advanced to a physiological and geometric reading of living matter.

Conclusion

Geometry and science were, for Leonardo da Vinci, the paths through which the human mind could rise toward the knowledge of divine laws. His approach was founded upon a requirement that was both moral and rational. Through reason and faith in a higher harmony, he sought to unveil the invisible forces that govern the universe.

Through the Vitruvian Man, he revealed that the beauty of the world proceeds from order, measure, and proportion, reflections of a divine harmony that the sages of Antiquity had already intuited. By uniting mathematics with the observation of reality, he achieved the most accomplished demonstration, that of an equilibrium in which man becomes the mirror and the measure of creation.

Thus, the Vitruvian Man does not only represent an ideal of proportions, but still today embodies the affirmation of a universal principle, that of a living unity that links the human body, nature, and the cosmos. Through this synthesis, Leonardo da Vinci stands as one of the most expansive minds of all time, at once scientist, artist, and philosopher, for whom observation, experience, and reason lead to the understanding of the supreme order of the universe.

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