Epistemological Contextualism from a Logical Point of View*

Contextualisme épistémologique d'un point de vue logique

Yves Bouchard[1]

[1] Université de Sherbrooke, Sherbrooke QC J1K2R1, Canada

yves.bouchard@usherbrooke.ca

RÉSUMÉ. Ce papier présente un cadre logique capable d'exprimer un contextualisme épistémologique. Le contextualisme épistémologique repose sur la possibilité d'une interprétation indexicale des connaissances de l'opérateur, selon laquelle les conditions de vérité des attributions des connaissances manifeste une variabilité contextuelle d'une telle manière que les connaissances dans un contexte n'implique pas de connaissances dans d'autres contextes. Au moyen d'une notion de contexte épistémique définie sur la base de la notion de contexte développée par McCarthy et Buvac en Intelligence Artificielle, le papier montre comment une interprétation indexicale des connaissances de l'opérateur peut être modélisée formellement à travers un système de déduction naturellequi permet le raisonnement classique parmi des contextes gouvernés par différents concepts de connaissance.

ABSTRACT. This paper aims at presenting a logical framework capable of expressing epistemological contextualism. Epistemological contextualism relies upon the possibility of an indexical interpretation of the knowledge operator, according to which the truth conditions of knowledge attributions exhibit a contextual variability in such a way that knowledge in one context does not entail knowledge in every context. By means of a notion of epistemic context defined on the basis of the notion of context developed by McCarthy and Buvač in artificial intelligence, I show how an indexical interpretation of the knowledge operator can be formally modeled through a natural deduction system that enables classical reasoning among contexts governed by different concepts of knowledge.

MOTS-CLÉS. Contextualisme épistémologique, contexte épistémique, normativité épistémique, raisonnement, logique, déduction naturelle.

KEYWORDS. Epistemological contextualism, epistemic context, epistemic normativity, reasoning, logic, natural deduction.

In this paper, I put forward a link between epistemology and artificial intelligence (AI), more specifically, a link between epistemological contextualism and the contextual logic of McCarthy and Buvač. I claim that the notion of context developed by McCarthy and Buvač [44, 14] in artificial intelligence can provide a formal framework for epistemological contextualism. In such a framework, different types of knowledge will have different properties. Some are more robust, others are more defeasible. As *types* of knowledge, logical knowledge, mathematical knowledge, scientific knowledge, empirical knowledge, testimonial knowledge, ordinary knowledge all exhibit significant differences in their epistemic qualification processes that involve different epistemic standards. The need for a pluralist view on knowledge stems precisely from the variety of epistemic standards used to qualify knowledge. As I intend to show, these different epistemic standards can be encapsulated in contexts in such a way that reasoning from one context to another will be formally regimented by epistemological rules.

In the first part, I present the conceptual framework of my proposal. I develop a notion of epistemic context and a notion of epistemological theory that are based on the contextual logic of McCarthy and Buvač. In the second part, I present a natural deduction system that incorporates all the key concepts of the first part, and that shows how reasoning through epistemic contexts can be achieved.

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1. Conceptual Framework

1.1. Two Problems: Indexicality and Context Shifting

Epistemological contextualism defends that knowledge attributions, i.e., assertions that are prefixed with a knowledge operator, require an indexical interpretation. In that view, the meaning of *I know that* ϕ is dependent on the context of assertion. One of the major challenges that epistemological contextualism is facing pertains to the clarification of the mechanisms at play in the indexical interpretation of the knowledge operator. This difficulty is not exclusive to contextualism, but it belongs to indexicality as a semantic theory and to the interpretation of any linguistic item of indexical nature.[1] The fact that what is at stake for contextualism is, in particular, the knowledge operator should not have any direct bearing on the semantic theory as such. This idea is the basis of an argument developed by Davis [22] to confront indexical contextualism. Davis contrasts the behavior of other indexical expressions with the behavior of knowledge claims interpreted indexically in order to show that, contrary to other indexical items, knowledge claims generate skeptical paradoxes. So, according to him, resorting to an indexical interpretation of knowledge claims does not provide any solution to the problems raised by the skeptical argument.[2] Even though I do not share every point of Davis' analysis, I definitely agree with him on the idea that if the indexical interpretation of knowledge claims is to be correct, then the knowledge operator should behave (generally) like any other indexical expression. The problem of indexicality for epistemological contextualism consists in showing how the reference of the knowledge operator, as in the case of any indexical expression, is contextually fixed with precision and how it is possible to disambiguate its meaning by means of its indexical content.

Epistemological contextualism faces also an important related problem, essential to its methodological relevance: How are epistemic contextual shifts regulated? If contextualism is to be contributive to epistemology, it must explain the dynamics within which our epistemic transactions are taking place. For instance, Lewis [43, 42] considers that shifts from one epistemic context to another are governed by accommodation rules that determine a conversational score for each state in a conversational context. Lewis' rules tend to favor the development of a conversational game by accommodating (i.e., presupposing the truth of) each conversational move as much as possible. This kinematics however conceals a significant drawback. Contextual accommodations are *automatically* made, i.e., when a new alternative (say a skeptical one) has been called into play, as far-fetched this alternative might be, the epistemic standard is at once raised to accommodate this alternative, consequently knowledge is harder to gain.[3] Lewis' perspective does not account for the autonomy of epistemic contexts with respect to epistemic agents, and this autonomy is one important issue for the representation of what is going on when a context shift

1.A high degree of generalization of contextual dependence may give rise to fine-grained distinctions. For example, Bianchi [4] distinguishes several forms of contextual dependence, following Perry and Searle, among which indexicality is only one case. Besides, Bianchi and Vassallo [5] even defend the idea that epistemological contextualism might help in clarifying the semantic thesis, if the theory of meaning includes a notion of justification.

2.Blaauw [6] has developed another line of attack against the indexical interpretation of knowledge attributions by conceiving K either as a scalar predicate (like *tall*) or as a pointer predicate (like *here*). As for me, in section 2, I will take the knowledge operator as a success term.

3.This is why knowledge tends to be elusive, according to Lewis [42]. The more an epistemic context prompts uneliminated (or uneliminable) possibilities of error, the less there is knowledge. So, the process of context shifting has for limit nothing less than impossible knowledge. For other contextualists, the demand put on context shifting is rather a matter of degree, as Cohen [19] underlined: "Though skeptical considerations frequently lead to a strong upward pressure on the standards, the shift to a skeptical context is not inevitable. The pressure toward higher standards can sometimes be resisted" (93).

takes place. If epistemic standards are entirely enslaved to characteristic fluctuations of conversational contexts, then there is no possibility of representing the properties of epistemic contexts otherwise than provisionally fixed by alternatives (or counterfactual situations) that are relevant to the conversational context *hic et nunc*. In my perspective, an epistemic context should be robust enough to allow any relevant hypothesis, even those that compromise the current or given epistemic framework, to be subjected to the *very same epistemic standard*, and this presupposes that the mere mention of a remote alternative, for instance, does not entail a context shift (from a strictly epistemological point of view, at least). Suffice it to say for the moment that the clarification of the constraints governing epistemic context shifts is of tantamount importance for the contextualist response to the problem of qualifying epistemic normativity.

1.2. Contexts in Epistemology and in AI

Contextualists like Lewis [42], Cohen [21, 18] and DeRose [23, 24] have presented their epistemological options in the guise of a response to skeptical arguments. I believe that this has not favored the elaboration of a notion of epistemic context that is explicit and well defined. Instead of being analyzed in the conceptual foreground, this notion has been kept in a more or less intuitive form and instrumentalized to the higher goal of a counteroffensive to skepticism. The proposed analysis here will follow a different direction and the notion of epistemic context will be given priority, so that it will be at the center of the epistemological investigation. The notion of epistemic context that will be developed in this section is inspired by several elements already in the literature, especially in the field of artificial intelligence where it has been a research object since the 90'. My claim is that, by means of an explicit characterization of this notion of epistemic context, it is possible to formulate an adequate solution to both the problem of indexicality and the problem of context shifting.

Since the seminal work of McCarthy [44, 45] on contexts, there has been many theories of context, as Brézillon [12] showed in his comprehensive survey. This is not the place to discuss all of these views. I just want to point at three salient conceptions of context in order to contrast the present proposal. A context may be conceived as an abstract object encoded in a formal language by means of an operator. More specifically, in McCarthy [44], Buvač, Buvač and Mason [16, 15], and Buvač [14], the `ist` modality is interpreted as truth in a model (or validity).[4] In that view, $\mathtt{ist}(c, \phi)$ means that a formula ϕ is true (or valid, after some generalization) in context c, i.e., ϕ is true in all truth assignments of context c. Buvač and Mason [14, 16, 15] have extended McCarthy's intuitions to modal logic. Under another interpretation, centered around meaning in natural languages, a context can be envisaged as a situation, i.e., a formal representation of factual dimensions [1]. One interesting feature of these two views on context is that they both provide means for lexical disambiguation. According to a third view, a context is a "*subset of the complete state* of an *individual* that is used for reasoning about a given goal" [30]. These states are belief states, hence a context is a set of an agent's beliefs. From a broader perspective, several semantics have been used to model contextual reasoning, for instance dynamic semantics [2, 3] and local models semantics [27].[5] All of theses conceptions have their advantages. But, in the following section, I will focus on the first of theses conceptions since it is the most abstract.

4.It may be interpreted more weakly as truth depending on the kind of constraints imposed on the model.
5.For clarifying comparisons of formal theories of context, see [11, 10, 48].

1.3. Contextual Logic of McCarthy and Buvač

My proposal rests on the idea that the contextual logic of McCarthy and Buvač (CL_{MCB}) can provide an adequate framework for the logical treatment of contextual knowledge. The key advantage offered by CL_{MCB} for epistemological contextualism consists in the fact that it allows for a complete expression of the knowledge operator as a triadic relation, i.e., a relation between an agent, a propositional content, and an epistemic standard. By making the standard explicit in the representation, it becomes possible to define explicitly the meaning of each epistemic operator and to study the properties of the relations among several epistemic operators. I will use the notion of *epistemic context* in order to incorporate these two aspects. As we shall see, an epistemic context will define the meaning of an epistemic operator (solution to the problem of indexicality), and it will also define the rules for intercontextual relations (solution to the problem of context shifting).

The notion of context and the contextual logic originally developed by McCarthy in the field of artificial intelligence aim at providing a solution to the problem of generality, i.e., the problem of representing ordinary knowledge and its integration to inferential processes operating on knowledge bases. CL_{MCB} can be defined generally as $FOL \cup \{ist(c, \phi)\}$, where FOL is classical first-order logic and $ist(c, \phi)$, an operator meaning that the formula ϕ is true in context c. The operator ist expresses a relation between a formula and a set of first-order true formulas which is reified as a formal object, a context. In CL_{MCB}, the completeness of FOL is preserved [17, 15], and even though this contextual logic is not strictly speaking an epistemic logic, comparable for instance to Lemmon [41] or Hintikka [40, 39], it can be nonetheless represented in a standard multimodal logic [15].

Buvač [14] defined the syntax of CL_{MCB} by means of the following axioms and rules[6]:

(PL) $\vdash_k \phi$, where ϕ is an instance of a propositional tautology

(UI) $\vdash_k (\forall x)\phi(x) \supset \phi(a)$

(MP) $\dfrac{\vdash_k \phi \quad \vdash_k \phi \supset \psi}{\vdash_k \psi}$

(UG) $\dfrac{\vdash_k \phi \supset \psi(x)}{\vdash_k \phi \supset (\forall y)\psi(y)}$, where x is not free in ϕ

(K) $\vdash_k ist(k', \phi \supset \psi) \supset (ist(k', \phi) \supset ist(k', \psi))$

(D) $\vdash_k ist(k_1, ist(k_2, \phi) \vee \psi) \supset ist(k_1, ist(k_2, \phi)) \vee ist(k_1, \psi)$[7]

(Flat) $\vdash_k ist(k_2, ist(k_1, \phi)) \supset ist(k_1, \phi)$

(Enter) $\dfrac{\vdash_{k'} ist(k, \phi)}{\vdash_k \phi}$

(Exit) $\dfrac{\vdash_k \phi}{\vdash_{k'} ist(k, \phi)}$

(BF) $\vdash_k (\forall v)ist(k', \phi) \supset ist(k', (\forall v)\phi)$

6.In the place of $\vdash k : \phi$, I simply use $\vdash_k \phi$ to mean that a formula ϕ is provable (or assertable) in the context k.

7.Buvač used Δ instead of D to refer to this propositional property of contexts. I shall use D in order to avoid any confusion with the usual symbol for knowledge bases, Δ.

The first group (PL, UI, MP, UG) comprises axioms and typical rules of FOL. In the second group ($K, D, Flat, Enter, Exit$), the axioms and rules express propositional properties of contexts; axiom K is a principle of deductive closure (an analogue of the axiom K in modal logic), axiom D (that Buvač called *contextual omniscience*) permits the qualification of any information accessible from any given context, axiom $Unif$ is a principle of information preservation through contexts, and the rules $Enter$ and $Exit$ permit to access or to leave a context. Finally, in the group of quantificational properties of contexts, there is one axiom (BF) analog to the Barcan formula specifying the relation between the ist operator and the universal quantifier.

Buvač [14] made a distinction between two classes of contexts, the *knowledge base* contexts (c_{kb}) and the *discourse* contexts (c_d). Whereas in c_{kb} predicates are univocal, in c_d predicates may be ambiguous. A c_{kb} is a set of true propositions, or facts, in a given knowledge base. A c_d is characterized by two components, a set of *epistemic states* and a set of *semantic states*. In an epistemic state, one finds typical elements of a knowledge base, i.e., facts. A semantic state sets the interpretation of a predicate by means of a relation to another predicate in a knowledge base. It is by virtue of such a relation that an ambiguous predicate in a c_d can be disambiguated.

The main motivation behind CL_{MCB} consists precisely in providing a formal framework for eliminating ambiguity.[8] This is where CL_{MCB} presents a special interest for epistemology. Since the knowledge operator has to be interpreted as an indexical term, according to epistemological contextualism, it is an operator that requires disambiguation in function of its context of utterance, and by the same token, an epistemic context has to be conceived as a c_d. In this view, CL_{MCB} can shed light on the dynamics at play between the interpretation of the knowledge operator and the epistemic contexts of utterance.

1.4. Epistemic Contexts

Taking advantage of CL_{MCB}, it is possible to define a notion of epistemic context that will exhibit formal properties. The notion of epistemic context (c_ε) that I will be using lies upon the idea that *an epistemic context c is a context defined by an epistemic standard ε that is an introduction rule for the knowledge operator in c.* In CL_{MCB} terms, the standard ε is a subset of the axioms of the knowledge base of c (Δ_c), and to each epistemic context c_ε is associated one and only one epistemic standard. Since it is the epistemic context that determines the meaning of the knowledge operator, then an epistemic context can be envisioned as a c_d, i.e., $\varepsilon \subseteq \Delta_{c_d}$ and more specifically $\varepsilon \subseteq SemanticStates(\Delta_{c_d})$ because ε provides the *indexical content* (variable part) of the meaning of the knowledge operator. In accordance with CL_{MCB}, the complete characterization of an epistemic context depends on a twofold characterization: a characterization of its *epistemic standard* (ε) and (if any) a characterization of its *transfer rules* (τ)[9], which are the rules that govern its relations with other c_ε.

These conceptual choices place at the center of the investigation the conditions for context shifting and, by way of consequence, the conditions for epistemic standard shifting. This is in line with the contextualist goal of accounting, on the one hand, for the dynamics observable in our epistemic exchanges, that express the variability of the epistemic standards in use, and on the other hand, for the legitimacy of these

8.It can also be extended to other types of contexts [35].
9.These rules were formerly called *transposition rules* in [8].

variations (i.e., they are not epistemic faults).[10] These variations in the use of epistemic standards show clearly our capacity as epistemic agents to regiment our epistemic practices accordingly to a plurality of norms in function of our epistemic needs.

One immediate consequence of the above definition of c_ε is that it entails a relativization of all contexts, including logical contexts, that is to say logical contexts are local epistemic contexts like any other epistemic contexts. This rises a difficulty of representation in CL_{MCB}, since CL_{MCB} has been devised with the explicit goal of making available logical reasoning in local contexts (via $lifting$) by means of a grammar incorporating FOL. The rules PL, UI, MP and UG render accessible the resources of FOL in every local context. However, this structure cannot account entirely for contextualism, because from the contextualist point of view FOL is only one epistemic context among others, and one can imagine that in some rich and complex epistemic situations many logics, stronger or weaker than FOL, may be called upon. Consequently, CL_{MCB} has to be amended in order to reify FOL so as to become an object of the language, which in turn requires the conversion of the rules PL, UI, MP, UG, K, and D into properties of epistemic contexts defined by logical standards.

Before considering some examples of epistemic contexts, I want to underline that the whole idea here is to give some insight into this notion of epistemic context, and my formalism will depart slightly from CL_{MCB} in that I make an explicit distinction among axioms between epistemic standards and transfer rules. By definition, an epistemic context will require one and only one epistemic standard, and most of CL_{MCB} grammatical rules (PL, UI, MP, UG, K, D) will be directly incorporated in contextual transfer rules. As a toy example of a set of epistemic contexts, consider the following three partial and plausible definitions of some ordinary (and common) epistemic contexts, $c_{logical}$, $c_{empirical}$ and $c_{perceptual}$:

Axioms of $c_{logical}$ (c_{log})

($\varepsilon_{log}.1$) $(\forall x)(\phi \supset K(x, \phi))$, where ϕ is an instance of a propositional tautology or of a first-order valid formula

($\tau_{log}.1$) $ist(c_{log}, \phi \supset \psi(x)) \supset ist(c_{log}, \phi \supset \forall y \phi(y))$, where x is not free in ϕ

($\tau_{log}.2$) $(\forall x)((ist(c_{log}, ist(c, K(x, \phi))) \wedge ist(c_{log}, ist(c, K(x, \phi \supset \psi)))) \supset (ist(c_{log}, ist(c, K(x, \psi)))))$

($\tau_{log}.3$) $(\forall x)(ist(c_{log}, ist(c, K(x, \phi \supset \psi))) \supset (ist(c_{log}, ist(c, K(x, \phi))) \supset ist(c_{log}, ist(c, K(x, \psi)))))$

c_{log} corresponds to the classical system of FOL. The axiom $\varepsilon_{log}.1$ is the epistemic standard defining c_{log} and it means that any instance of a propositional tautology or of a valid formula of FOL is sufficient for knowledge.[11] $\tau_{log}.1$, $\tau_{log}.2$, and $\tau_{log}.3$ are respectively the syntactic rules UG, MP, and K of CL_{MCB} expressed in terms of rules of transfer. It is worth noticing that $\tau_{log}.2$ guarantees reasoning by *modus ponens* within the scope of the knowledge operator in a given and fixed context, in the very same manner $\tau_{log}.3$ preserves deductive closure in a logical context.[12] According to the formulation of $\tau_{log}.3$, the

10.In that regard, and contrary to what Schiffer [47] suggested, contextualism does not need an error theory to accommodate an indexical interpretation of knowledge attributions.

11.One will recognize in $\varepsilon_{log}.1$ an analogue to the rule of necessitation in modal logic.

12.$\tau_{log}.3$ is comparable to a principle of scope alteration, that switches the scope of K (superior level) with the one of $\supset$ (inferior level). Such a permutation is tolerable solely in a logical order.

epistemic context c of the antecedent and of the consequent remain fixed.[13] Finally, one can see that any valid pattern of inference can be expressed in the form of a rule of transfer and the set of these rules could be ultimately reduced to a single axiom schema.

Axiom of $c_{empirical}$ (c_{emp})

$(\varepsilon_{emp}.1)$ $(\forall x)(EmpiricalControl(x, \phi) \supset K(x, \phi))$

$\varepsilon_{emp}.1$ stipulates that the condition to satisfy in order to introduce the knowledge operator in this context is some sort of empirical control made by an agent x with respect to the state of affairs described by a proposition ϕ. The notion of empirical control in $\varepsilon_{emp}.1$ consists only in a set of procedures providing a sufficient level of discrimination between a state of affairs described by a proposition ϕ and a state of affairs described by a proposition (or several propositions) incompatible with ϕ. In the present illustration, no transfer rule enables one to export empirical knowledge to another c_{ε}.

Axiom of $c_{perceptual}$ (c_{per})

$(\varepsilon_{per}.1)$ $(\forall xv)((See(x, v) \vee Hear(x, v) \vee Taste(x, v) \vee Smell(x, v) \vee Touch(x, v)) \supset K(x, \phi))$, where ϕ is immediately linked to v

As regards the perceptual standard, things are different since v is not a propositional content but rather a perceptual content. The knowledge operator is introduced only in virtue of a perceptual state (or a percept). The knowledge operator is in this way dependent on our physiological mechanisms and their respective limitations (think of the various perceptual biases identified by cognitive psychology for instance). No transfer rule is available in c_{per}.

The fact that neither c_{emp} nor c_{per} comprise a transfer rule is determined exclusively by the definitions of the epistemic standards. A transfer rule makes possible the propagation of knowledge either within a given context or between different contexts. As opposed to the grammatical rules $Enter$ and $Exit$ which are only rules of access to information, the transfer rules act as qualification rules in much the same manner epistemic standards themselves do. The transfer rules of c_{log} ($\tau_{log}.1$, $\tau_{log}.2$, $\tau_{log}.3$, and $\tau_{log}.4$) are intracontextual rules of transfer. No such rule has been defined in c_{emp} and c_{per} for simplicity concern. Furthermore, there is no intercontextual rule of transfer for $\{c_{log}, c_{emp}, c_{per}\}$ in reason of the epistemic standards in use. In c_{per}, for instance, the assertability conditions are evidently too weak to satisfy the assertability conditions of c_{log} and c_{emp}. There is no intercontextual rule of transfer between c_{per} and c_{emp}, because the satisfaction of ε_{per} does not imply the satisfaction of ε_{emp} (ε_{per} is simply too weak), and conversely, the satisfaction of ε_{emp} does not entail the satisfaction of ε_{per} (a property, for example, may be tested empirically while not being itself an object of direct perception). This shows clearly the primitive character of the notion of epistemic standard, which dictates the possibility or the non-possibility of transfer rules. As for the question whether a transfer rule can be valid *a priori*, i.e., independently of any epistemic standard, one can easily see its irrelevance within the proposed contextualist framework.

Another noticeable aspect of the previous definitions is that no intracontextual rule of transfer specifies the conditions of transmission of a knowledge item from one epistemic agent to another. One could think,

13.As I will show in section 3.3., failures of deductive closure take their origin essentially in a confusion between distinct epistemic contexts.

for instance, that if an agent a has run an empirical control with respect to ϕ and $K(a, \phi)$, then an agent b, who knows that a has performed a test, would know by some testimonial relation that ϕ. More formally: if $\vdash_{c_{emp}} K(a, \phi)$ and $\vdash_{c_{emp}} K(b, K(a, \phi))$, then $\vdash_{c_{emp}} K(b, \phi)$. The main difficulty in the formulation $\vdash_{c_{emp}} K(b, K(a, \phi))$ can be straightforwardly isolated. If b knows that $K(a, \phi)$, then it is surely not in virtue of ε_{emp} since b is not the one who has run the test, but in virtue of another epistemic standard, namely $\varepsilon_{testimony}$. The specification of all the transfer rules for testimonial knowledge constitutes a major issue from an epistemological point of view. These rules require a fine-grained analysis that goes well off the limits of the present paper. Given that my treatment aims only at presenting a workable notion of epistemic context, it is preferable on this occasion to omit the problem of knowledge transmission between agents.

1.5. Epistemological Theory

The perceptual standard, the empirical standard, and the logical standard defined above give a view on the ordinary epistemic resources at our disposal as epistemic agents. But the chief interest of the toy example lies elsewhere. In defining epistemic contexts by means of explicit epistemic standards, one not only gives the knowledge operator its various meanings, but one also describes a structure in which epistemic normativity is spelled out in different terms. Such a conception of epistemic normativity allows for multiple configurations of epistemic contexts, which in turn can be captured by the idea that *an epistemological theory is as a set of* c_ε. The epistemological theory presented above, say Θ, is defined as $\Theta = \{c_{log}, c_{emp}, c_{per}\}$. An epistemological theory is consequently defined by a specific set of epistemic contexts (or knowledge bases), that is to say a specific set of epistemic standards and transfer rules. The epistemological structure of the theory is given by the transfer rules that govern the inter and intracontextual relations between contexts. This definition provides a new perspective upon major debates in contemporary epistemology. Foundationalism, coherentism, reliabilism, and other options based on the JTB model, may be construed as exemplifying different epistemological structures designed to meet different epistemic demands. None of them is the ultimate epistemological theory simply because all of them are instances of particular structural configurations.

The specific structure of an epistemological theory shows the relations between the different assertability conditions of the knowledge operator proper to each context. It could seem that my treatment of epistemic normativity is eluding the crucial problem of the truth conditions of the knowledge operator. Of course, behind this difficulty is lying the debate between a realist and an antirealist interpretation of the knowledge operator. In the proposed view, the truth conditions of K in a given epistemic context are provided by the assertability conditions of K in the given context, so that truth-conduciveness from one context to another follows assertability from one context to another. The purpose of a transfer rule is to authorize the dissemination of assertions in multiple contexts on the basis of one given context. The function of transfer rules though is to be sharply distinguished form the function of the ist operator, because the formula in the argument position of the operator is in mention not in use. The $Exit$ rule makes explicit the genealogy, so to speak, of the truth of a formula from another context, whereas the $Enter$ does exactly the inverse, i.e., it encapsulates the truth in the assertability conditions of a context.

For a realist, this isomorphic relation between truth conditions and assertability conditions boils down to the elimination of the truth conditions, conceived as contextually independent.[14]

These considerations lead naturally to another important difficulty that a contextualist perspective is facing. Can contextualism account for the implication between knowledge and truth, as the factivity (or veridicality) condition requires it, i.e., $K\phi \supset \phi$? This time the debate takes place between a fallibilist and an infallibilist conception of knowledge.[15] The factivity condition springs from an analysis centered on the necessary conditions for knowledge (analysis *in consequentia*). The framework developed here makes explicit only the sufficient conditions for knowledge (analysis *in antecedentia*); the epistemic standards are nothing else than mere introduction rules for the knowledge operator, and the antecedent of the epistemic standard may not even contain any epistemic terms, depending on the context. As I take it, here lies the main interest of contextualism as it constitutes a general epistemological framework within which epistemic normativity can be analyzed primarily in terms of its function rather than its content. So, in order to make explicit the characterization of some K by means of necessary conditions, the general contextualist framework has to be singularized and this process amounts to the specification of an epistemological theory, as previously defined.

According to the proposed framework, and in conformity with McCarthy and Buvač [45], the epistemic contexts are conceived independently from the epistemic agents. This only means that the epistemic perspective of one given agent does not alter in any way the facts, or the epistemic states of Δ_{c_ε}. This property of *flatness* makes it easier to isolate the contextual variations at the level of the contexts themselves, in other words at the level of their respective transfer rules. This reification of an epistemic context brings autonomy to the context with respect to the epistemic agents, and this accounts for the constraint that within one given epistemic context all of the epistemic agents are regimented by the very same epistemic standard and submitted to the very same epistemic demands. Certainly one could define an epistemic context with a parameter in relation with the propositional attitudes of the epistemic agents so that a context would vary in function of the agents. But such a change would represent more than a change of epistemological theory, it would be more radically a change of logic (or grammar) since one would have to give up the $Unif$ axiom of CL_{MCB} in order to render possible alterations of the epistemic states of one context by means of another context. No doubt the rejection of $Unif$ would be relevant in some particular epistemological investigations, but within the limits of the proposed approach that would have the undesirable effect of concealing (at least partially) the dynamics between the epistemic standards that precisely I want to isolate.

1.6. Skeptical Argument Revisited

As an illustration, let us apply the proposed framework to an exemplary epistemological problem such as a skeptical argument. Consider Dretske's [25] visit at the zoo. In this scenario, the skeptic is claiming that the visitor, who presumably knows that the animals in the pen marked "Zebras" are zebras, cannot

14.Some realists, like Williamson [52, 53], go as far in the opposite direction as making knowledge the norm of assertion. In my view, such a reversal in the assertability conditions does not make justice to the observable variability of epistemic standards in our epistemic practices.

15.It is instructive to notice that Lewis' contextualism is grounded on an infallibilist conception of knowledge [42]. In the epistemological theory Θ presented above, ε_{per} shows a high level of fallibility, compared to ε_{emp}, which is moderated, and to ε_{log} which is null.

know that these animals are really zebras since she does not know whether or not these animals are cleverly disguised mules. In other words, the visitor cannot eliminate a possibility of error that jeopardizes her knowledge. In order to rebut the visitor's knowledge (or *alleged* knowledge, so would say the skeptic), the skeptic logically derives a contradiction out of the visitor's knowledge, using an epistemic closure principle (knowledge under entailment). Several distinct epistemic contexts are at play in this argument and the sketch of an epistemological theory characterized above can make them explicit.

This skeptical argument articulates three epistemic facts: (1) the visitor knows (or presumably knows) that the animals in the pen are zebras, (2) the visitor knows that if an animal is a zebra then this animal is not a cleverly disguised mule, but (3) the visitor does not know whether the animals in the pen are cleverly disguised mules. The primary difficulty here is ambiguity. The epistemic contexts that determine the meaning of the knowledge operator are entirely inhibited, which suggests (wrongly) that the meaning of K remains constant in each knowledge attribution. A correct analysis of this problem demands first a disambiguation of the meaning of K by means of its indexical content, i.e., the epistemic standard used to qualify knowledge in a given epistemic context. Let's say that p stands for *the animals in the pen are zebras*, q for *the animals in the pen are cleverly disguised mules*, and $\vdash_k \phi$ for ϕ *is assertable (or asserted) in context k*, then one can express in Θ the epistemic facts in the following manner:

(1) $\vdash_{c_{per}} K(a,p)$

(2) $\vdash_{c_{emp}} K(a, p \supset \neg q)$

(3) $\vdash_{c_{emp}} \neg K(a, \neg q)$

The assertion 1 means that the visitor (a) knows that p in virtue of ε_{per}. It could not be otherwise, because in the situation of an ordinary visit at the zoo a is not in a position to satisfy ε_{emp} of Θ. The formula 2 asserts that a knows empirically, i.e., according to ε_{emp}, that $p \supset \neg q$. It is also clear that a could not know perceptually that $p \supset \neg q$, since ε_{per} does not allow for (perceptual) discrimination between p and q.[16] The third epistemic fact 3 expresses the empirical ignorance of a with respect to $\neg q$, given that a has not performed any empirical test in the situation.[17] Up to this point, I believe there is nothing controversial in this $\Theta-$representation of the epistemic facts proper to the argument *à la Dretske*.

The contradiction the skeptic wants to obtain can be derived either in the perceptual context or in the empirical context. In the first case, the skeptic will need $\vdash_{c_{per}} \neg K(a,p)$ (strategy 1), which would contradict 1, and in the second case, the skeptic will need $\vdash_{c_{emp}} K(a, \neg q)$ (strategy 2), which would contradict 3. In order to represent the logical reasoning of the skeptic on the information available in c_{per} and c_{emp}, we first need to export the epistemic facts 1-3 to the logical context, c_{log}:

(4) $\vdash_{c_{log}} ist(c_{per}, K(a,p)) \qquad Exit, 1$

(5) $\vdash_{c_{log}} ist(c_{emp}, K(a, p \supset \neg q)) \qquad Exit, 2$

(6) $\vdash_{c_{log}} ist(c_{emp}, \neg K(a, \neg q)) \qquad Exit, 3$

16.This situation is akin to the one evoked by Goldman [32] concerning fake barn façades.

17.Of course, in an epistemological theory other than Θ the expression of these same epistemic facts could differ significantly. The point of my argument though does not consist in promoting one particular epistemological theory, but rather to put into light the contextualist dynamics present in any epistemological theory.

Now, the desired contradiction may be obtained, according to strategy 1, by using closure ($\tau_{log}.3$) and by adding, for the sake of the example, a contraposition rule ($\tau_{log}.4$) to c_{log}:

(7) $\vdash_{c_{log}} ist(c_{emp}, K(a,p)) \supset ist(c_{emp}, K(a,\neg q))$ $\quad \tau_{log}.3$, 5

(8) $\vdash_{c_{log}} ist(c_{emp}, \neg K(a,\neg q)) \supset ist(c_{emp}, \neg K(a,p))$ $\quad \tau_{log}.4$, 7

(9) $\vdash_{c_{log}} ist(c_{emp}, \neg K(a,p))$ $\quad \tau_{log}.2$, 6,8[18]

(10) $\vdash_{c_{emp}} \neg K(a,p)$ $\quad Enter$, 9

The logical development 7-9 conforms with the parameters of the situation, for the visitor cannot know according to ε_{emp} that the animals in the pen are zebras (9) because she has not performed any empirical test. Assertion 10 expresses this logical result in the empirical context. At this stage of the argument, it is important to stress the point that 10 does not prevent the visitor from knowing according to ε_{per} that the animal in the pen are zebras (1). And there is no contradiction (nor incompatibility) between $\vdash_{c_{emp}} \neg K(a,p)$ and $\vdash_{c_{per}} K(a,p)$. The only way the skeptic can produce a contradiction in c_{per} is by transferring 10 to c_{per}:

(11) $\vdash_{c_{per}} \neg K(a,p)$ $\quad ?$

Assertion 11 exhibits the critical move, that relies upon a transfer from $\vdash_{c_{emp}} \neg K(a,p)$ to $\vdash_{c_{per}} \neg K(a,p)$ (11). With 11, the skeptic would show that Δ_{per} of a is inconsistent in reason of $\vdash_{c_{per}} K(a,p)$ and $\vdash_{c_{per}} \neg K(a,p)$. The strategy 1 can be diagrammed in this way (where the dotted lines represent the litigious transfers):

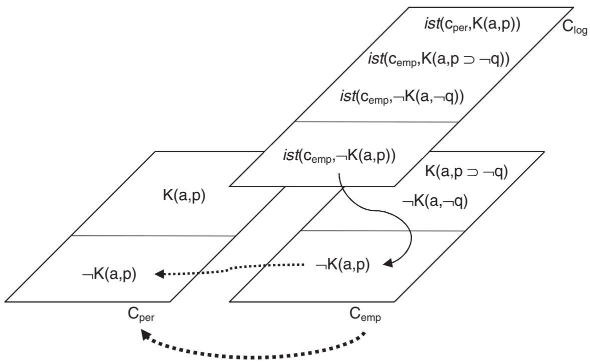

Figure 1. Strategy 1

The validity of the transfer (11) between c_{emp} and c_{per} (dotted lines) however cannot be simply presupposed. In Θ, the relation between the standards ε_{per} and ε_{emp} does not allow for a transfer from c_{emp} to c_{per}, thus the passage from $\vdash_{c_{emp}} \neg K(a,p)$ to $\vdash_{c_{per}} \neg K(a,p)$ is not permitted. So, this strategy of the skeptic rests upon a false presupposition with respect to the possibility of a transfer.

According to the second strategy, the skeptic wants to construct a contradiction in the empirical context with $\vdash_{c_{emp}} K(a,\neg q)$. This can be achieved by means of an initial transfer from $\vdash_{c_{per}} K(a,p)$ to $\vdash_{c_{emp}} K(a,p)$ and *modus ponens*:

18.The skeptic could also reason by *modus tollens*, but that would conceal the resort to the principle of deductive closure since the required rule of transfer would be: $(\forall x)((ist(c_{log}, ist(c, \neg K(x,\psi))) \wedge ist(c_{log}, ist(c, K(x, \phi \supset \psi)))) \supset (ist(c_{log}, ist(c, \neg K(x,\phi)))))$.

(12) $\vdash_{c_{emp}} K(a,p)$?

(13) $\vdash_{c_{log}} ist(c_{emp}, K(a,p))$ *Exit*, 12

(14) $\vdash_{c_{log}} ist(c_{emp}, K(a,\neg q))$ τ_{log}.2, 13,7

(15) $\vdash_{c_{emp}} K(a,\neg q)$ *Enter*, 14

The strategy 2 can be diagrammed in this way:

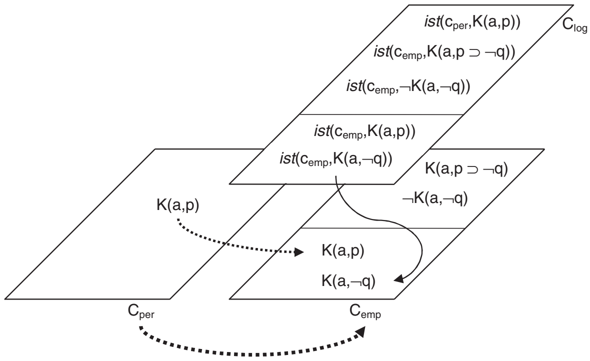

Figure 2. Strategy 2

Here again the skeptic presupposes that one can transfer $\vdash_{c_{per}} K(a,p)$ to $\vdash_{c_{emp}} K(a,p)$ and, by logic, one can obtain a contradiction with $\vdash_{c_{emp}} \neg K(a,\neg q)$ (3) and $\vdash_{c_{emp}} K(a,\neg q)$ (15). Yet such a transfer is only possible if the relation between ε_{per} and ε_{emp} allows for it, which is not the case in Θ (as it was for strategy 1). The skeptical argument demands a rule of transfer like $(\forall x)(ist(c_{per}, K(x,\phi)) \supset ist(c_{emp}, K(x,\phi)))$, with or without its converse.[19] In the absence of such a rule, the skeptical argument is plainly invalid.

The preceding contextualist analysis shows that the explicitation of the properties of epistemic contexts enables one to cope with the skeptical argument in a satisfactory manner. The contextualist model explains not only why the skeptical argument cannot be generalized, but also why, for instance, deductive closure is not at fault. Within the contextualist framework, transgression of transfer rules between epistemic contexts are made entirely explicit. In the particular case of the skeptical argument, a context shift is taking place tacitly.[20] In the proposed framework, all epistemic qualifications are made explicit, so that it is not possible to shift from one context to another tacitly. In a given epistemological theory, all of the transfer rules are defined so to preserve the designated epistemic property (namely K) through context shifting, whenever it is possible.[21]

2. Natural Deduction System

In this part, I develop a natural deduction system that encompasses all the key concepts of the proposed conceptual framework. The goal of such a system is to provide a proof-theoretic approach with respect to knowledge, within which epistemic normativity can be logically investigated.

19.If the skeptic wants to produce a contradiction also in the perceptual context, with $\vdash_{c_{per}} K(a,\neg q)$, the needed transfer rule must be stronger, namely $(\forall x)(ist(c_{per}, K(x,\phi)) \equiv ist(c_{emp}, K(x,\phi)))$.

20.Cohen's [20] diagnosis turns out to be totally correct: "The apparent closure failures are illusions that result from inattention to contextual shifts" (111). Hendricks [38] underlines rightly that the closure principle is valid in a fixed context.

21.Furthermore, this notion of transfer rule dispenses one with the usual considerations regarding the notion of *relevant alternative* understood in counterfactual terms, as in [25, 49, 46, 20, 37].

The development of a natural deduction system for contextual reasoning has been undertaken by Giunchiglia [30, 31] in the perspective of avoiding recourse to modal logic. This resulted in a system described as *multilanguage hierarchical logics*, or multilanguage systems (ML), not only capable of embedding modal system K for instance, but also richer that standard modal systems. Compared to such a project, my goal is very modest. I want to make logically explicit the qualification process of epistemic items. This requires that I define (1) the conditions under which knowledge operators are introduced in the first place, and (2) how knowledge instances can be propagated through contexts.

The notion of epistemic context will respond to requisite (1), and the notion of epistemological theory will respond to requisite (2). As in section 1.4., an epistemic context is defined by a unique epistemic standard. A knowledge context (K-context) is an abstract entity that associates a knowledge operator (K-operator) to an epistemic standard. Generally, a K-context is totally defined by the set of all the formulas prefixed with the corresponding K-operator. The semantic rule for K is given by

$$[\![K_i\phi]\!] = \text{true iff } \vdash_i \phi,$$

where $\vdash_i \phi$ means any wff ϕ derivable in K-context i. This provides for an indexical interpretation of the K-operator which makes possible in turn the representation of a plurality of knowledge concepts. The epistemic differences between K-operators are always explicit in the representation as they are indexed to their associated epistemic context. A consequence of this reading is that truth is synonymous with satisfaction of a standard.

An epistemological theory Θ is defined by a pair $\langle\kappa, \tau\rangle$, where κ is a family of K-contexts and τ is a set of transfer rules between K-contexts. Transfer rules are here conceived as axioms rather than inferential schemas in order to make epistemic qualification through contexts variable in the system. In this way, an epistemological theory specifies the epistemic relations among a set of K-contexts. The characterization of these relations are of tantamount importance, as Guha and McCarthy [35] have underlined:

> "If the statements ϕ and $\phi \Rightarrow \beta$ have different contextual dependencies, the program [inference engine] can't always combine them to conclude β. Before combining two sentences with different contextual dependencies, the program needs to reconcile relative contextual dependencies. [...] To cope with the contextuality, it [the program] needs to be able to factor out the relative contextuality so that it can use knowledge gathered in one context in another." (166)

So, in the formal representation, not only the contextual dependencies of assertions will be made explicit, but the relations between contexts will also be explicit. The proposed framework satisfies these demands.

2.1. Rules

For the purpose of simplicity, I will restrict the deductive resources to classical propositional logic. I want to center the attention around the particular behavior of the K-operator.[22] I will follow a Fitch-style presentation for the natural deduction system to emphasize graphically the relation of conditionality between epistemic contexts (or derivations).

22.For a generalized approach allowing for different sets of deductive schemas, see [27, 29, 31].

Two rules of reiteration are distinguished for formulas prefixed and not prefixed with a K-operator:

$$\begin{array}{l|l}
\vdots & \vdots \\
m & \phi \\
\vdots & \vdots \\ \hline
\vdots & \vdots \\
n & \phi \qquad \text{R}, m
\end{array}
\qquad\qquad
\begin{array}{l|ll}
\vdots & \vdots & \\
m & K_i\phi & \\
\vdots & \vdots & \\ \hline
\vdots & \vdots & \\
n & i & \vdots \\
o & & K_i\phi \qquad \text{KR}, m
\end{array}$$

Any hypothesis can be reiterated in the same context (R). This is a restricted form of the usual natural deduction rule. As for the other rule (KR), it states that a hypothesis of type $K_i\phi$, and only a hypothesis of this type, can be reiterated in a K-context i (KR).

Introduction (KI) and elimination (KE) of the K-operator satisfy the constraint (and the conception) that a derivation is associated with a K-context:

$$\begin{array}{l|ll}
1 & \phi_1 & \\
\vdots & \vdots & \\
m & \phi_m & \\ \hline
\vdots & \vdots & \\
n & i & \vdots \\
o & & \psi \\
o+1 & K_i\psi & \qquad \text{KI}, n\text{–}o
\end{array}
\qquad\qquad
\begin{array}{l|l|l}
\vdots & i & \vdots \\
m & & K_i\phi \\
\vdots & & \vdots \\
n & & \phi \qquad \text{KE}, m
\end{array}$$

provided ψ is not a formula
prefixed with K_i

KI expresses the idea that the formula ψ satisfies the epistemic standard which defines the K-context i (provided ψ is not prefixed with K_i). The rule prohibits the construction of formulas of type $K_iK_i\psi$, because it is incoherent to apply directly any standard to itself.[23] The rule KE conforms with the principle of factivity of knowledge, as long as truth is understood as assertability in a K-context.

There is also a rule for the negation of the K-operator ($\neg$KI), which behaves like the usual negation:

$$\begin{array}{l|ll}
1 & \phi_1 & \\
\vdots & \vdots & \\
m & \phi_m & \\ \hline
\vdots & \vdots & \\
n & i & \underline{\psi} \\
\vdots & & \vdots \\
o & & \omega \\
\vdots & & \vdots \\
p & & \neg\omega \\
p+1 & \neg K_i\psi & \qquad \neg\text{KI}, n\text{–}p
\end{array}$$

23.It is incoherent in the sense that, for example, the length of the *standard meter* cannot be used to measure the length of the standard meter itself.

$\neg K_i\psi$ does not mean to be *ignorant* with respect to ψ. Ignorance is a property of an agent. Since a K-operator refers to a K-context, which is defined by a unique epistemic standard, then $\neg K_i\psi$ means that ψ cannot be known in K-context i; ψ cannot be asserted in accordance with the epistemic standard specific to K-context i. And, $\neg K_i\psi$ is different from $K_i\neg\psi$, for the latter formula can only be the result of an application of KI.

Finally, there is a special rule for transferring knowledge from one context to another, which is a kind of KR by means of an axiom:

$$\begin{array}{l|llll}
\vdots & \vdots & & & \\
m & K_i\phi & & & \\
\vdots & \vdots & & & \\
n & j & \multicolumn{1}{|l}{\vdots} & & \\
o & & \multicolumn{1}{|l}{K_j\phi} & & \mathrm{KT}_{i-j}, m
\end{array}$$

A formula of type $K_i\phi$ can be transferred in a K-context j, where $i \neq j$, by means of a transfer rule $i-j$ such that $KT_{i-j} : K_i\phi \supset K_j\phi$. The KT rule acts as a kind of reiteration rule through epistemic contexts on the basis of a conditional epistemic qualification. Transfer rules, when available, are provided by the epistemological theory, and they determine the configuration of K-contexts.[24]

3. Discussion

In order to stress a few peculiarities of the proposed system, I will address some epistemological issues. In this section, I (1) highlight the importance of transfer rules, (2) examine the KK-thesis (introspection), and (3) analyze the closure principle (under known entailment).

3.1. Transfer Rules

In the proposed perspective, an epistemological theory is fundamentally specified by a set of transfer rules. For instance, if one compares the following epistemological theories $\Theta_1 - \Theta_3$, two of which are determined by one single different transfer rule τ with respect to the main context K_0 (implicit) and an arbitrary context *i*, then one gets deductions in Table 1. In the epistemological theory Θ_1, for which no transfer rule is defined ($\tau = \varnothing$), none of the five deductions are valid. Such an epistemological theory is strongly restrictive, since all K-contexts are isolated one from the other. At the opposite end of the spectrum, a theory Θ_4 defined as being equal to $\Theta_2 \cup \Theta_3$ would trivialize the notion of epistemic context, since all K-contexts would be equivalent to each other.[25] This shows clearly the impact of transfer rules on the way information is accessed and exploited throughout epistemic contexts.

The present framework exhibits some similarities with ML, but there are also differences. One of these differences pertains to transfer rules. According to Giunchiglia [30], a context encapsulates an agent's (subjective) perspective about the world, a kind of *point of view*. The relations that take place between

24.Since the definition of a transfer rule is expressed in terms of material implication, a configuration of K-contexts is a partially ordered structure. Hence, it is not possible to have a transfer rule between two (logically) incompatible K-contexts.

25.The difference between Θ_1 and Θ_4 echoes the distinction between modal systems K and $S5$ with respect to the accessibility relation.

Θ_1	Θ_2	Θ_3
$\tau = \varnothing$	$\tau = KT_{i-0}$	$\tau = KT_{0-i}$
$K_i p \nvdash p$	$K_i p \vdash p$	$K_i p \nvdash p$
$p \nvdash K_i p$	$p \nvdash K_i p$	$p \vdash K_i p$
$\nvdash K_i p \supset p$	$\vdash K_i p \supset p$	$\nvdash K_i p \supset p$
$\nvdash p \supset K_i p$	$\nvdash p \supset K_i p$	$\vdash p \supset K_i p$
$\{K_i p, (p \supset q)\} \nvdash K_i q$	$\{K_i p, (p \supset q)\} \vdash K_i q$	$\{K_i p, (p \supset q)\} \vdash K_i q$

Tableau 1. Comparison of three epistemological theories

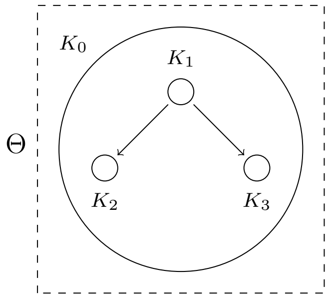

Figure 3. Epistemological theory Θ

contexts, as point of views, are conceived in a multiagent perspective, and the propagation of knowledge is made possible by means of what Giunchiglia calls *bridge rules*.[26] One such rule is the *multicontextual* version of modus ponens (MMP):

$$\text{MMP}: \quad \frac{\langle \phi \supset \psi, i \rangle \qquad \langle \phi, j \rangle}{\langle \psi, k \rangle}$$

The MMP rule allows an agent k to believe a propositional formula ψ, given the beliefs that $\phi \supset \psi$ and that ϕ of agents i and j respectively. This bridge rule permits a fusion between contexts, i.e., between sets of agents' beliefs. In ML, a bridge rule is an inference schema, and it requires, as such, a logical account. But, in the proposed framework, logic alone is not sufficient to warrant this kind of intercontextual relations. The epistemological notion of transfer rule conveys precisely the idea that there is a change in *epistemic qualification* involved in the process, as opposed to sheer reiteration for instance. So, epistemic qualification rules are properly understood as constituents of an epistemological theory, and they demand an epistemological account. Of course, this is a matter of theoretical choice. But the rationale for the present framework is the focus put on epistemic qualification, i.e., on epistemic normativity. In that view, this framework is more of a norm-based framework than an agent-based framework.

One virtue of MMP is that it warrants the propagation of knowledge among agents considered as epistemic peers (knowledge bases, for instance). But when a group of epistemic agents is rather heterogeneous and the modalities of knowledge acquisition (or K-contexts) in the group are numerous and not equivalent, then variation in the epistemic qualification processes must be allowed.

For instance, consider an epistemological theory Θ comprised of four K-contexts, where K_0 is the main logical context (tacit), K_1 is an expert context, and K_2 and K_3 are two sorts of mundane contexts,

26.Bridge rules are different, for instance, from the *transition rules* of Gonzalez [34, 33]. For Gonzalez [34], transition rules (or sentinel rules) "contain the transition criteria that cause the currently active major context to deactivate itself and activate another more relevant major context" (827). They behave like production rules.

equipped with a single transfer rule $KT_{1-i} : K_1\phi \supset K_i\phi$, as in Figure 3 In such an epistemic situation, Θ would permit theses instances of MMP,

$$\frac{\langle \phi \supset \psi, 1\rangle \quad \langle \phi, 2\rangle}{\langle \psi, 2\rangle}\,\text{MMP} \qquad \frac{\langle \phi \supset \psi, 1\rangle \quad \langle \phi, 3\rangle}{\langle \psi, 3\rangle}\,\text{MMP}$$

but it would not allow neither of the following deductions:

$$\frac{\langle \phi \supset \psi, 1\rangle \quad \langle \phi, 2\rangle}{\langle \psi, 3\rangle} \qquad \frac{\langle \phi \supset \psi, 1\rangle \quad \langle \phi, 3\rangle}{\langle \psi, 2\rangle}$$

In general, if an epistemological theory does not assume a homogeneous process of epistemic qualification (i.e., a unique epistemic standard), then the contingency of MMP is no surprise. So, this is one way to represent the heterogeneity and richness of an epistemological theory. Such a pluralism in the representation of epistemic normativity not only provides an adequate model for epistemic situations involving expert and profane modes of knowledge acquisition, but it also amounts to make room for *testimonial* knowledge in the representation. And, sure enough, the representation of testimonial knowledge is a major challenge for a formal system.

The proposed system differs from Giunchiglia's ML in that a K-context cannot be assimilated to the set of an agent's beliefs, unless the agent is an idealization of a unique epistemic standard. MMP, as an inference schema, embodies some kind of an idealized relation between epistemic agents; it requires a univocal reading of the knowledge operator. But, from an epistemological point of view, an agent's knowledge base is the result of the application of many epistemic standards, which have different epistemic strength. The preference to use axioms rather than inference rules, that would impact on the structure of the system from a proof-theoretic perspective, becomes manifest in that an inference schema expresses a metaproperty of the whole deduction system, but an axiom is rather an object-language property particular to an epistemological theory.

Ghidini and Serafini [29] have proposed a significant extension of bridge rules that provides more generality. In their system, i.e., distributed first-order logic (DFOL), bridge rules are enriched with cross-domain variables. And one important property of DFOL consists in the understanding of a bridge rule as a distributed relation of logical consequence.[27] But, here again, some differences between transfer rules and bridge rules lie in the fact that knowledge is a modality (iterable) and that transfer rules are not strictly assimilable to logical consequence. A transfer rule defines a relation of epistemic qualification between two epistemic standards, which is expressed as an axiom in terms of material implication, not in terms of logical consequence. Transfer rules are stipulated in the object-language; they are not part of the metalinguistic machinery *stricto sensu*, as opposed to bridge rules. So, one can see a difference between transfer rules and DFOL bridge rules not so much in the results of their application, but rather in the *level* according to which they are imposing their constraints on the deductive system.[28]

27.In that regard, DFOL shares some properties with labeled deduction systems (LDS) [26]. Differences between DFOL and LDS are presented in [28].

28.There might be some way to capture transfer rules by means of bridge rules that I am overlooking, but at this point in time it is not clear to me at all how to encode the former into the latter. A difficulty is particularly apparent in the case of second-order knowledge. For instance, assume a transfer rule such as $KT_{1-2} : K_1\phi \supset K_2\phi$ for which $\phi \coloneqq K_3(p)$ in K_1. Then, in K_2, one would get $K_3(p)$. How would this translate in terms of a bridge rule?

3.2. Introspection

Let us put the proposed deduction system at work and appreciate how it can express and deal with some epistemological issues. Among the derivable formulas of an epistemological theory that includes three K-contexts, without transfer rules (like Θ_1), one has

$$K_1(p \supset q) \vdash K_1 p \supset K_1 q \tag{D}$$

$$\{K_1 p, K_1(p \supset q)\} \vdash K_1 q \tag{C}$$

$$K_1 p \vdash K_2 K_1 p \tag{I}$$

These formulas express some properties of K-operators with respect to material implication and to iteration. Formula (D) is analogous to axiom K in modal logic, i.e., the K-operator distributes over material implication. In the epistemological literature, some of these formulas are litigious from an epistemological point of view, namely the formulas expressing closure (C) and introspection (I).

Consider introspection (I) first. Introspection means that knowledge iterates, i.e., if one knows that ϕ then one knows that one knows that ϕ, or formally $K\phi \supset KK\phi$. Usually, introspection is understood as being an analogue of axiom 4 in modal logic, i.e., $\Box\,\phi \supset \Box\,\Box\,\phi$. The main difficulty with such a reading, from a strict epistemological standpoint, lies in the fact that no distinction is made between iterated K-operators. In other words, iterations of K-operators are conceived *univocally*. This interpretation is typical of epistemological theories that rely on a concept of knowledge expounded in representational terms (cognition). But, a univocal interpretation conceals some epistemic discrepancies. In the case of perception, for example, a univocal interpretation of the introspection principle implies a perceptual impossibility, as long as an agent a cannot see herself seeing a book, in a univocal sense *i*: $See^a_i(book) \supset See^a_i(See^a_i(book))$. On the other hand, there is no a priori difficulty in conceiving that $See^a_i(book) \supset See^a_j(See^a_i(book))$ when $i \neq j$. One can see in a *cognitive* sense that one sees in a *perceptual* sense. From an epistemological standpoint, a univocal reading of the K-operator also jeopardizes the idea that an epistemic agent must be in a position to assess her own epistemic process. But, if an agent cannot discriminate between a situation where she does satisfy an epistemic standard from a situation where she does not, then this agent does not exhibit epistemic agency. The capability of assessing epistemic processes (or satisfaction of standards) is a necessary condition for epistemic agency. When a theory conflates knowledge and cognition, the KK-thesis, just like a univocal reading of the K-operator, faces the very same difficulty as in the perceptual case. In the present view, it is possible to escape this difficulty insofar knowledge is conceived as a *qualified* form of cognition, in that it is the result of the satisfaction of an epistemic standard, and applicability of epistemic standards is assessable.

In this framework, prefixed formulas of the form $K_i K_i \phi$ are prohibited by the KI rule. But formulas of the form $K_i\phi \supset K_j\phi K_i\phi$, when $j \neq i$, are straightforwardly deducible:

1			$K_1 p$	
2	*2*	*1*	$K_1 p$	KR, 1
3			p	KE, 2
4			$K_1 p$	KI, 2–3
5			$K_2 K_1 p$	KI, 2–4
6			$K_1 p \supset K_2 K_1 p$	⊃I, 1–5

So, *heterogeneous* forms ($j \neq i$) of the KK-principle are vindicated, but *homogeneous* forms ($j = i$) are invalidated. This is in line with the idea that in order to be able to apply an epistemic standard, an epistemic agent needs to be able to evaluate *epistemologically* the result of its application; when one knows something, *one knows that she has satisfied some epistemic standard*. More generally, and this is what the framework expresses, any K-context can be part of a broader K-context.[29]

3.3. Closure

The skeptical argument analyzed in section 1.6. can be easily represented now. Here again, part of the problem stems from a lack of explicit specification of the K-operators involved in the principle. As stated in (C), the closure principle requires that the K-operators in the premises and in the conclusion are the same:

1	$K_1 p$		
2	$K_1(p \supset q)$		
3	*1*	$K_1 p$	KR, 1
4		p	KE, 3
5		$K_1(p \supset q)$	KR, 2
6		$p \supset q$	KE, 5
7		q	$\supset$E, 4, 6
8	$K_1 q$		KI, 3–7

In such a case, closure holds, as the current epistemological theory shows. Let us call this form of closure, *intracontextual* closure. In the case where several K-operators are involved in the reasoning, then the situation is quite different, and this form of closure is an *intercontextual* one. An agent might accept that she knows that p, and that she knows that p is incompatible with q, *while denying* that she knows that $\neg q$. The crux of the problem in this sort of argument is the difference between types of knowledge: there is a difference between knowledge resulting from perceiving the animals in the pen, and knowledge resulting from conceiving the incompatibility between two properties (being a zebra, and being a cleverly disguised mule). In this view, the principle that is challenged by the argument, in this particular instance, is not intracontextual closure, but rather intercontextual closure. Consequently, the problem should be reframed accordingly, and the two K-operators K_1 and K_2 could be interpreted, for illustrative purpose, as referring respectively to perceptual knowledge and to reflective (or some kind of abstract) knowledge. In the present framework, the deduction would be represented this way:

1	$K_1 p$		
2	$K_2(p \supset \neg q)$		
3	*1*	$K_1 p$	KR, 1
4		p	KE, 3
5		$K_1(p \supset \neg q)$	? (requires KT_{2-1})
6		$p \supset \neg q$	KE, 5
7		$\neg q$	$\supset$E, 4, 6
8	$K_1 \neg q$		KI, 3–7

29.There is a connection in that regard with what McCarthy [44] referred to in terms of *transcending contexts* by means of lifting.

Without a transfer rule between the different epistemic contexts involved in the reasoning (line 5), there is no way to preserve closure between contexts.[30] So, intercontextual closure is not a valid principle in an epistemological theory that does not contain a transfer rule like KT_{2-1}. But, this fact does in no way alter the validity of intracontextual closure. So, it is possible, in the same epistemological theory (like the one under consideration) to have intracontextual closure without intercontextual closure. It is worth stressing that a total rejection of closure has the devastating consequence of compromising the expansion of knowledge, not only between different K-contexts (inter), but also within the same K-contexts (intra). And, one lesson we can draw from this difficulty is that an adequate analysis of the closure principle, both from a logical point of view and an epistemological point of view, demands an epistemological framework capable of expressing differences between K-operators (or epistemic standards).

4. Conclusion

Let us have a new look at the two initial problems for contextualism. Within the perspective of epistemological contextualism, the main issue consists in accounting for the variations in the meaning of K-operators in function of the variations of epistemic contexts while preserving part of the meaning throughout these variations, and all of this can be achieved by means of an indexical interpretation. This is precisely what the conceptual framework provided by CL_{MCB} can model and clarify. By defining an epistemic context as a context regimented by a unique epistemic standard, the application of a K-operator is thereby oriented by this precise parameter which fixes its indexical content. It then becomes possible to disambiguate the various uses of K-operators.

But, as one might object, this structure is by far too idealized to represent all the richness and all the complexity of our epistemic practices. I am surely not claiming that epistemic agents should behave in the same way knowledge base systems work, and that the former should be reduced or even conformed to the latter. My claim is rather that CL_{MCB} makes explicit what is constitutive of our ordinary and very complicated epistemic practices, namely that we use K-operators in accordance with some contextual determinants and these determinants vary from context to context. An informative way to analyze these determinants is to conceive of them as introduction rules that give K-operators their contextual meaning. It is this process that can be captured and abstracted by a knowledge base system as CL_{MCB} and the above deduction system. The main interest of such an abstraction is the clear view it provides on epistemic normativity, a view that is too easily obscured by *a priori* philosophical considerations. As an object of inquiry, epistemic normativity emerges from the manifold of epistemic practices. Only once the mechanisms by which a set of epistemic conditions may transform into an epistemic norm have been explicited can one appreciate the density and the fitness of our ordinary epistemic practices. The variations observable at the higher level of abstraction (indexicality) are the decanted parts of the variations observable at the basic level of the epistemic practices. This is no surprise since we are responsible for our epistemic practices, and even though some of them are very robust and some of them are very weak (depending on the epistemological theory at stake), in any case epistemic agency is what makes them all possible as, ultimately, an adapted response to our environment.

30.In that perspective, Dretske's argument, which amounts to deny intracontextual closure by means of a failure of intercontextual closure, does not hold and the problematic fragment of the reasoning is made explicit in the framework.

The problem of context shifting receives a direct solution insofar all contextual changes are regulated by transfer rules that proceed from the epistemic standards defining the contexts. The content of the transfer rules determines explicitly the possible relations between epistemic contexts, so that illicit contextual shifts can be identified with precision. Illicit context shifts are sometimes very subtle but often well meshed into the fabric of natural language, and the confusion they generate takes frequently the form of a philosophical aporia, as the debates around deductive closure (among others) show.[31] The application of the principle of deductive closure requires that the destination-context of the conclusion be the same as the source-context of the premises. When a skeptical argument, for instance, exhibits a failure in deductive closure, it indicates clearly that a context shift has taken place tacitly. In this regard, a failure of deductive closure can even serve as a reliable indicator of a tacit context shift.[32]

The goal of an epistemological theory consists in making explicit the rules that govern the use of K-operators (epistemic standards), and the rules that govern the possible relations between epistemic contexts (transfer rules). The distinction between epistemic standards and transfer rules confers to the theory more expressivity, and to the epistemic standards, more autonomy. Context shifting involves both aspects of the epistemological theory. And one important virtue of the contextualist framework is that it can contribute to the clarification of the conditions under which this process of context shifting can take place.

The proposed framework is based on the idea that knowledge can be conceived as the satisfaction of an epistemic standard. Such a characterization, in terms of a successful application of rules (epistemic norms), can be adequately captured by a proof-theoretic approach, and this is the motivation behind the development of a natural deduction system in which knowledge is expressed by means of K-operators and K-contexts. It provides a logical framework for the investigation of epistemological theories and their respective properties. Hence, this framework enables a pluralist view on knowledge.

This kind of epistemological pluralism might appear to some as an untenable stand. I agree that this conception of an epistemological theory as being a structure of K-contexts augmented by some relations is quite an abstraction. But, in my view, the chief interest of such an abstraction lies in its expressive power with respect to the (observable) variety of epistemic norms which are characteristic not only of our most robust scientific endeavors, but also of our most ordinary epistemic practices. In a laboratory situation, experimenters articulate several knowledge acquisition means, such as perception, measurements, statistics, logic, and so on. In an ordinary situation, epistemic agents articulate knowledge acquisition means like personal experience, history, reflection, testimony, and so on. It is my main claim that these complex epistemic situations reflect various epistemological theories, and that a deductive system capable of expressing variations in epistemic normativity is interestingly informative about knowledge, and how we, as a species, use it to cope with our environment.

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31.In addition to the usual skeptical paradoxes, one can think of the paradox of confirmation, the Gettier problems of type II, the lottery and the preface paradoxes.

32.This is a view I developed in Bouchard [7]. For other views on the failure of deductive closure, see Brueckner [13], Vogel [50], Hales [36], and Warfield [51].

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