Paradox of entropy: a bit of information without energy

Paradoxe de l'entropie : un bit d'information sans énergie

Version 2 [1]

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ABSTRACT. Physical systems materializing a bit of memory have recently been the subject of experiments to explore the link between information and entropy or to realize Maxwell's demons. Whether microscopic (single electron, molecule) or mesoscopic (glass ball, colloidal particle, nano-magnet) systems, their changes of state are the result of elementary transitions caused by thermodynamic fluctuations or by quantum tunnelling.
From the probabilistic equations of these transitions we deduce directly the formulas of energy exchanges between the memory, the energy source which controls its state and the thermostat which surrounds it, avoiding a detour by the theory of information. The resulting theory explains all the experimental results of bistable and tilt memories. It shows the effect of the speed of operations on the energies involved and on the heat dissipation.
It brings to light the paradox of the temporality of entropy, which had been evoked by Landau and Lifschitz, namely that entropy is not independent of time, more precisely that the value of entropy at this energy scale depends on the duration retained for its evaluation. It explains why it is possible in some cases to create or delete a bit of information with an energy well below $Kb\, T\, log\, 2$, the Landauer limit, and it solves the enigma of the Szilard's machine.

RÉSUMÉ. Des systèmes physiques matérialisant un bit d'information ont récemment fait l'objet d'expériences pour explorer le lien entre l'information et l'entropie ou pour réaliser des démons de Maxwell. Qu'il s'agisse de systèmes microscopiques (électron unique, molécule) ou mésoscopiques (bille de verre, particule colloïdale, nano-aimant), leurs changements d'état sont le résultat de transitions élémentaires provoquées par les fluctuations thermodynamiques ou par un effet tunnel quantique.
Les équations probabilistes de ces transitions permettent d'établir directement les formules d'échange d'énergie entre la mémoire, la source d'énergie qui en contrôle l'état et le thermostat qui l'entoure, en évitant un détour par la théorie de l'information. La théorie qui en résulte explique l'ensemble des résultats expérimentaux des mémoires bistables et des mémoires à bascule. Elle montre l'effet de la vitesse du processus sur les énergies en jeu et sur la dissipation thermique.
Elle met au jour le paradoxe de la temporalité de l'entropie, qui avait été évoqué par Landau et Lifshitz, à savoir que l'entropie n'est pas indépendante du temps, plus précisément que la valeur de l'entropie à cette échelle d'énergie dépend de la durée retenue pour son évaluation. Elle explique pourquoi il est possible de créer ou d'effacer un bit d'information avec une énergie largement inférieure à $Kb\, T\, log\, 2$, la limite de Landauer, et elle résout complètement l'énigme de la machine de Szilard.

KEY WORDS. Entropy, information theory, bistable memory, tilt memory, Maxwell's demon, Landauer 's limit, Szilard's machine, entropy temporality.

MOTS-CLÉS. Entropie, théorie de l'information, mémoire bistable, mémoire à bascule, démon de Maxwell, limite de Landauer, machine de Szilard, temporalité de l'entropie.

1. Introduction

In recent years, several teams of researchers have carried out experiments on physical systems with two states, either to try to prove Landauer's principle, according to which the erasure of a bit of information requires an energy dissipation of at least $k_B T \ \log 2$ (Boltzmann's constant $k_B = 1.38\ 10^{-23} J/K$) at temperature T, or to show that this same amount of energy can be extracted from a bit of information. These types of systems include bistable memory, consisting of two potential wells separated by a modular barrier, or tilt memories, whose two states are separated by a modular energy difference. They operate at the energy level of thermodynamic fluctuations ($k_B T$, approximately 1 to 400 10^{-23} J depending on the experiment).

[1] This version 2 uses more realistic formulas for the transition frequencies, without consequences on the conclusions of the study.

Some types of bistable memory use a nano-object subjected to gravity, whose state changes are caused by an electric field ([DiLu09] [JuGB14] [RMPP14] [GaBe16]) or a viscous force ([BAPC12]). Others use a nano-magnet, in which a component of the magnetic field determines the height of the potential barrier which separates the two states, and in which the perpendicular component of the field causes the changes of state ([HLDB16] [GBMZ17]). Tilt memories use a single-electron box controlled by an electric field ([KMPA14]), or a molecule with two stable configurations controlled by a mechanical force ([RiRi19]). In bistable memory, one control parameter regulates the height of the barrier which separates the two states. For both types of memory, an actuator, which can be mechanical, electrical or magnetic, enables varying the potential difference between the two possible states.

The authors of these experiments model heat transfers or the work provided by the actuator based on Shannon's information theory ([Shan48]). To my knowledge, no one has proposed a theoretical model explaining all phenomena involved during state changes in these types of memory. By avoiding the information theory approach, the equations are obtained directly for quasi-static processes, and dynamic processes can be simulated using formulas giving the transition frequencies from one state to another, which only depend on the energy difference between the states.

In a previous article [Argo21] the author demonstrated that Landauer's limit does not apply to tilt memories, since these make it possible to reversibly invert a bit, and therefore to erase it, in the sense of Landauer, without energy dissipation. However, it remained to explain in detail the elementary phenomena which cause the changes of states of these memories, in particular for mesoscopic systems. He thus proposed the hypothesis of underlying quantum phenomena.

The present study shows that we can forgo this hypothesis and that the laws of statistical thermodynamics are sufficient to explain all of these processes. The proposed model directly applies to single-electron, bistable-molecule systems and the single-molecule Szilard virtual machine ([Szil29]). It provides an explicit solution for the quasi-static transformations and a numerical simulation for the faster, non-equilibrium evolution.

The study reveals a paradox that appears at the energy level $Kb\,T$, namely that the value of entropy depends on the time scale considered, thus contradicting its classical definition $dS = dQ/T$. This temporality has, for instance, the consequence that one can create a bit of memory, with the corresponding reduction of entropy $Kb\ log\ 2$ without expenditure of energy [2].

2. Physical bit theory

2.1. *Two-state memory*

We consider a physical system made up of two potential levels, of energy U_0 and U_1, in which we can control the inversion from one state to the other, and ensure the stability over time of each state through an energy source, which will be called an *actuator*. This system is in contact with a thermostat maintained at a constant temperature T.

The energy difference between states 0 and 1 is $\Delta U = U_1 - U_0$, which becomes $\Delta U = U_1$ if we take U_0 as the origine of energy.

Subsequently, unless otherwise indicated, the quantities of internal energy U, thermal energy Q and work W supplied by the actuator are expressed in units $Kb\,T$, and the entropy in units Kb, since we

[2] This refers to the Boltzmann energy level $k_B T$, disregarding weaker limitations, such as the quantum limit, which results from the Heisenberg uncertainty relation ([GBMZ17])

seek to produce a memory whose energy is as low as possible, of the order of magnitude of thermal fluctuations, i.e. $Kb\,T$.

The energy difference is then expressed as: $\phi_e = \frac{\Delta U}{Kb\,T}$.

The general diagram of the memory is shown in Figure 1, which shows a state 0 of zero potential energy, a state 1 of potential energy ϕ_e, separated by a barrier of potential energy ϕ_e.

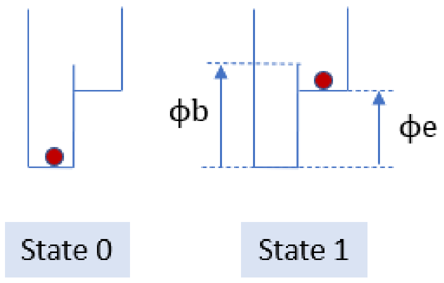

Figure 1. *Bi-state memory*

In bistable memory, the energy barrier ϕ_b separates the 0 and 1 states to stabilize them. In tilt memory there is no energy barrier ($\phi_b = \phi_e$) and the only controlling parameter is the energy gap ϕ_e. These memories have only two possible states, 0 or 1, without intermediate position. Transitions from one to the other are almost instantaneous, caused by a stochastic phenomenon, which can be of quantum nature (spin transition, tunnelling effect) or classical, by thermal fluctuations.

At equilibrium, ϕ_e and ϕ_b being constant, the probabilities P0 and P1 of the two states are given by the canonical partition function:

$$P_0 = \frac{e^{-\frac{U_0}{Kb\,T}}}{e^{-\frac{U_0}{Kb\,T}} + e^{-\frac{U_1}{Kb\,T}}} \qquad \text{hence :} \qquad P_0 = \frac{1}{1 + e^{-\phi_e}} \quad \text{et} \quad P_1 = \frac{1}{1+e^{\phi_e}} \qquad (1)$$

In the case where $\phi_e = \phi_b = 0$ the memory switches randomly from one state to the other according to the characteristic frequency f_0 of the system: its state is “randomized.” For memory to be usable, it must be stable for a certain period of time. This requires that the transition frequency from one state to the other be sufficiently low and therefore that ϕ_e or ϕ_b be sufficiently high.

When changing from one stable state to the other stable state, there may be several elementary transitions between the two states before stabilization in a final state.

2.2. *Elementary transitions*

Figure 2 represents an elementary transition from state 0 to state 1 in the case where $\phi_e > 0$. A thermal fluctuation or a tunnelling effect with an energy higher than ϕ_h, leads to a jump towards the state 1. This results in a change in potential energy ϕ_e, independent of ϕ_b. The height of the barrier ϕ_b acts only on the frequency of the transitions. In this case, the thermostat provides thermal energy δQ, which is transformed into potential energy δU, without modification of the control parameters ϕ_e and ϕ_b: $\delta Q = \delta U = \phi_e$.

Conversely, during a spontaneous transition from state 1 to state 0, the memory loses the potential energy $\delta U = -\phi_e$ transformed into thermal energy transmitted to the thermostat $\delta Q = \delta U = -\phi_e$.

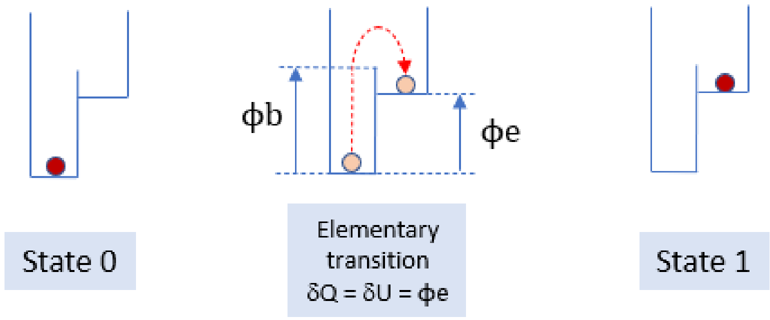

Figure 2. *An elementary state transition*

2.3. *Evolution of the state of the system.*

The elementary transition frequencies F_{01} (from 0 to 1) and F_{10} (from 1 to 0) depend on ϕ_e, ϕ_b and the energy profile of the system.

If we know these two characteristic functions, the initial state of the system as well as the evolution of $\phi_e$and $\phi_b$as functions of time, it is possible to calculate by iteration the state of the system at any moment. The process is stochastic, each elementary transition results in a discontinuity, and the energy balance is extremely variable (see section 3.3 below).

It is possible to explicitly solve the problem for a quasi-static process (whose speed tends to zero) and calculate the probability distribution of the state of the system at any moment.

P_0 and P_1being the probabilities of states 0 and 1, the probability P_1 varies according to the evolution equation ([Seki10] p.107):

$$\mathrm{d}P_1 = (P_0\, F_{01} - P_1\, F_{10})\, dt$$

or:

$$\mathrm{d}P_1 = \big((1 - P_1)F_{01} - P_1\, F_{10}\big)\, dt$$

In the case of a quasi-static evolution we observe the equilibrium:

$$P_0\, F_{01} = P_1\, F_{10}$$

From (1) we can deduce: $\frac{F_{01}}{F_{10}} = e^{-\phi_e}$ (2)

This equation remains applicable to non-equilibrium evolution, taking into account the quasi-instantaneous nature of elementary transitions[3].

2.4. *Energy transfers*

The potential energy of the memory is U_0 in state 0, with probability P_0 , and U_1 in state 1 with probability P_1. Its average value is therefore: $U = P_0\, U_0 + P_1\, U_1$.

With the conventions of section 2.1, this equation becomes: $U = P_1 \phi_e$

Hence the variation: $dU = \phi_e dP_1 + P_1\, d\phi_e$

The energy balance of the system implies: $dU = dQ + dW$

To a $d\phi e$ variation corresponds the work dW: $dW = P_1\, d\phi_e$ (3)

And, by difference: $dQ = \phi_e\, dP_1$ (4)

[3] This is confirmed experimentally in particular by [KMPA14] and [MMFH09].

2.5. *Thermal energy and entropy*

For a quasi-static evolution it is possible to integrate these differential equations to obtain an explicit solution.

According to (1): $$\phi_e = \log\frac{1-P_1}{P_1} \quad (5)$$

By combining (4) and (5): $$\frac{dQ}{dP_1} = \log\frac{1-P_1}{P_1}$$

By integration: $$Q(P_1) = -[P_1 \log P_1 + (1-P_1)\log(1-P_1)] + \text{constant}$$

Hence the thermal energy supplied to the memory between an initial state and a final state:

$$\Delta \boldsymbol{Q} = \left[-\mathbf{P_1} \log \boldsymbol{P_1} - (\mathbf{1}-\boldsymbol{P_1})\log(\mathbf{1}-\boldsymbol{P_1})\right]_{\boldsymbol{P_{1,initial}}}^{\boldsymbol{P_{1,final}}} \quad (6)$$

When randomizing a bit, i.e. when changing from $P_1 = 0$ or 1 to $P_1 = 0.5$, the memory receives thermal energy $\Delta Q = \log 2$.

This heat transfer causes a change in memory entropy: $\Delta S = \frac{\Delta Q}{T}$.

We see that the entropy of the memory decreases by $\log 2$ when passing from the random state ($P_1 = 0.5$) to a stable state ($P_1 = 0$ or 1), and is worth, according to P_1:

$$S(P_1) = -(P_1 \log P_1 + (1-P_1)\log(1-P_1))$$

We find there the formula $-(P_0 \log P_0 + P_1 \log P_1)$ proposed by Shannon to measure the quantity of information of a bit ([Shan48]).

But these variations ΔQ and ΔW are only average values obtained over a large number of experiments (see figure A1 in the Appendix), and successive experiments give extremely scattered results.

2.6. *Work of the actuator*

By differentiating (7) we obtain, at equilibrium: $$d\phi_e = -\frac{1}{P_1\,(1-P_1)}\,dP_1$$

and according to (3): $$dW = -\frac{1}{1-P_1}\,dP_1$$

we deduce, by integration: $$\mathbf{\Delta W} = [\,\mathbf{log}(\mathbf{1}-\boldsymbol{P_1})]_{\boldsymbol{P_{1,initial}}}^{\boldsymbol{P_{1,final}}} \quad (7)$$

Thus, to make the memory undergo a quasi-static evolution from its random state $P_1 = 0.5$ to the materialization of a bit (0 or 1), the actuator must provide, by an isothermal transformation, an energy $\Delta W = \log 2$ transmitted to the thermostat in the form of heat, and the reverse operation extracts this thermal energy from the thermostat to transmit it to the actuator in the form of work $\Delta W = \log 2$.

3. Application to two-state memories

3.1. *Bistable memory*

Consider a bistable memory in state 1 and in equilibrium ($\phi_e = 0, P_0 = P_1 = 0.5$). The energy level of the barrier ϕ_b is high enough to ensure its stability.

We want to invert the state of the memory (figure 3) by gradually increasing ϕ_e from 0 to a value ϕ_h greater than ϕ_b. When ϕ_e approaches or exceeds ϕ_b, a transition to state 0 occurs. Any reverse transition to state 1 is very unlikely, due to the high value of ϕ_b which opposes it. The process is asymmetrical. If $P_1 = 0$, the energy supplied to the system by the actuator is, by applying (7):

$$\Delta W = [\log(1 - P_1)]_{P_1 = 0.5}^{P_1 = 0} = \log 2$$

It remains to reduce ϕ_e to the value 0 to return to equilibrium, at state 0. The probability of an inverse transition is close to zero due to the high value of ϕ_b. The energy balance of this phase is therefore close to zero. Finally it was necessary to supply the energy $\Delta W = \log 2$ to the memory to invert the bit. The potential energy of the memory not having changed between the initial state and the final state, we conclude that the energy ΔW = log 2 has been transformed into heat. This operation is non-reversible, and the overall entropy of the memory and thermostat assembly has increased by log 2.

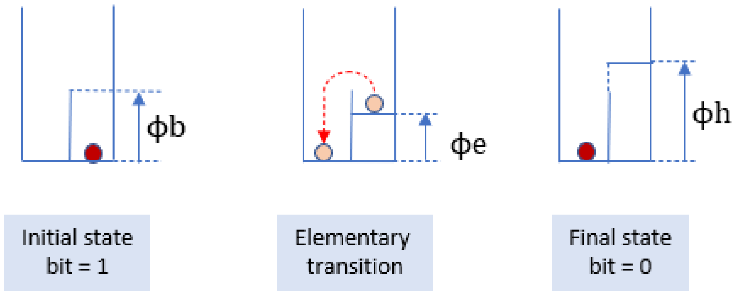

Figure 3. *Bistable memory*

Action on the height of the barrier

Let us assume again the memory at equilibrium (ϕ_e= 0) at state 1. If the height of the barrier is lowered from ϕ_b to 0, the elementary transitions which can occur during this operation are energy neutral, since they do not involve any variation in potential energy.

Randomizing a bistable memory bit is therefore energy neutral, contrary to Landauer's assertion that erasing a memory bit irreversibly dissipates an energy of at least $Kb\ T\ log\ 2$.

3.2. *Tilt Memory*

In tilt memory there is no potential barrier. Figure 4 shows a quasi-static transition from the random state P_1 = 0.5 to the final state P_1 = 0.

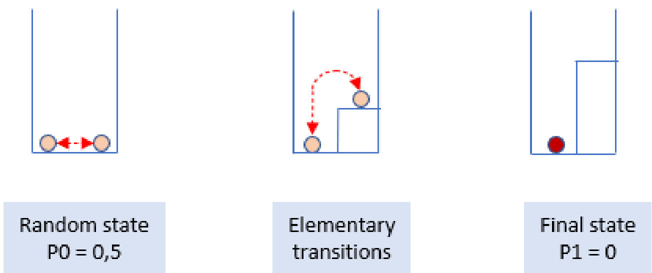

Figure 4. *Tilt Memory*

As for the bistable memory, the energy to be supplied is, by applying (7):

$$\Delta W = [\log(1 - P_1)]_{P_1 = 0.5}^{P_1 = 0}$$

That is:	$\Delta W = \log 2$
which corresponds to the variation in the thermal energy	$\Delta Q = -\log 2.$
and the decrease in entropy of the memory	$\Delta S = -\log 2$

Here the process is reversible. There is no overall increase in entropy, only reversible transfer of entropy between thermostat and memory.

3.3. *Non-equilibrium evolution of a tilt memory*

If the previous process represented in figure 4 is carried out quickly enough so that it is no longer quasi-static, we must return to the approach by elementary transitions of section 2.2. The exchange of energy and heat depend on the transition frequencies F_{01} and F_{10} between states, which themselves depend on the physical process in action.

The functions $F_{01}(\phi_e)$ and $F_{10}(\phi_e)$ were experimentally evaluated for a tilt memory based on the folding/unfolding of a DNA molecule ([MMFH09]) and for a single electron box ([KMPA14]) . In both cases the functions are symmetrical $\left(F_{01}(\phi_e) = F_{10}(-\phi_e)\right)$.

In the case of the DNA molecule used by [MMFH09], thsee functions are not only symmetrical, but also linear. These are straight lines which meet at the equilibrium point corresponding to the equilibrium frequency f_0 for $\phi_e = 0$

The only solution which also respects the constraint of equation (2) is in that case:

$$F_{01} = f_0 \, e^{-\frac{\phi_e}{2}} \quad \text{et} \quad F_{10} = f_0 \, e^{\frac{\phi_e}{2}} \qquad (8)$$

These formulas make it possible to calculate step by step the energy W to be supplied by the actuator and the heat Q received from the thermostat, as a function of ϕ_e, the temporal evolution of which is known. The evolution of ϕ_e due to the actuator requires an energy $\delta W = \delta \phi_e$ when the state is 1, and $\delta W = 0$ when the state is 0.

The probability of a transition during a time δt is ($F_{01}\ \delta t$) or ($F_{10}\ \delta t$) depending on whether the state is 0 or 1. We draw lots according to this probability to determine whether there is a transition or not.

If there is no transition, we have $\delta Q = 0$. Then if state is 1: $\delta U = \delta W = \delta \phi_e$, otherwise: $\delta U = 0$.

If there is a transition from state 0 to state 1, the memory receives from the thermostat a thermal energy $\delta Q = \phi_e$. So we get: $\delta U = \delta W + \delta Q = \phi_e$.

If there is a transition from state 1 to state 0, the memory supplies the thermostat with thermal energy $\delta Q = -\phi_e$. So we get: $\delta U = \delta \phi_e - \phi_e$.

The corresponding algorithm is reported in the Appendix.

Let us apply the method to the creation of a state 0 ($P_1 \simeq 1$) from the random state (P_1= 0.5), with the equilibrium frequency $f_0 = 1\ s^{-1}$ and a linear variation of the potential energy between 0 and ϕ_h = 8, i.e. P_1= 0.9997, by averaging 1000 simulations for each value of the cycle time tc.

For a quasi-static process (figure 5, for tc = 200 s), the simulation shows a value Q(t) centered on the function Qs(t) calculated according to formulas (3) and (6). The difference between the W and -Q curves corresponds to the potential energy stored in the memory during the process. The two values of W and -Q result in log 2 at the end of the cycle as computed in section 2.5.

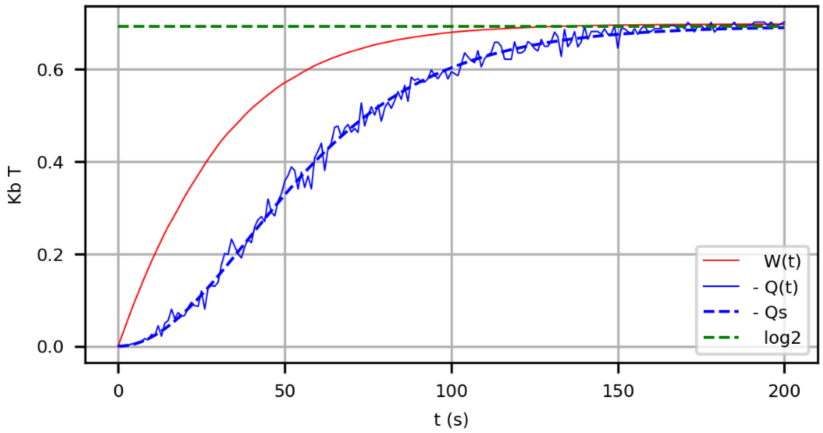

Figure 5. *Evolution of W and $-Q$ during quasi-static creation of a bit*

For a faster process (figure 6, for tc = 0.2 s), the final value of W = 2.7 exceeds log 2. The difference corresponds to the energy dissipated during the cycle. The final value of Q is less than the final value of W. This corresponds to cases where the final state is equal to 1. When this happens, after the end of the cycle, the state quickly drops to zero, the difference W + Q is transformed into heat and we get Q = -W.

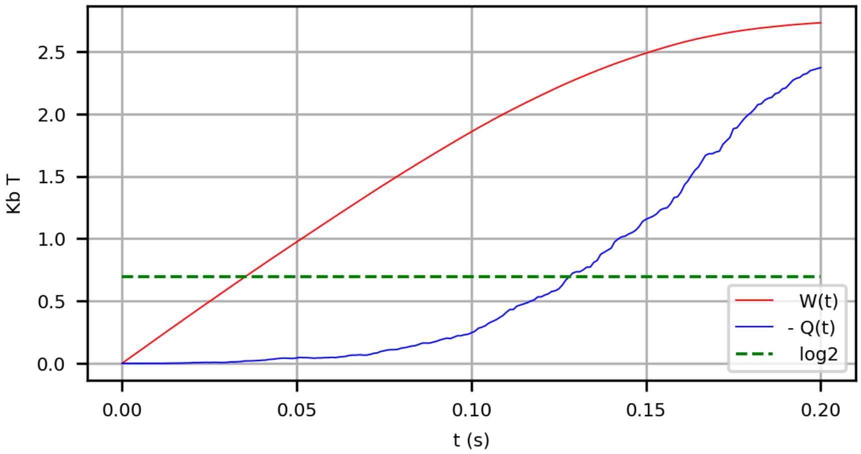

Figure 6. *Evolution of W and –Q during the creation of a bit for t_c = 0.2 s*

The evolution curves of the final values of W and -Q as a function of the cycle duration are shown in Figure 7. They indicate average values for 1000 simulations for each calculated point. Despite this large number of simulations, irregularities due to the stochasticity of the phenomena are still visible.

The curve (W, -Q) represents the energy W supplied by the actuator, which is transformed into thermal energy Q = - W at the end of the cycle, after the memory state has returned to zero. The right asymptote is reached for tc ~100 s for a quasi-static process.

The left asymptote for tc ~1 ms corresponds to a process that is sufficiently brief so that the state of the memory has not changed during the action. If the initial state was 0, we have W = Q = 0. If the initial state was 1 we have W = ϕ_h= 8 at the end of the cycle, but the state is very unstable and it drops very quickly to zero, this energy being transformed into heat. Since the initial state is equally distributed between 0 and 1, in the end we have on average W = - Q = $\phi_h/2$ = 4. The Qr curve in Figure 7 represents this energy transformed into heat when returning to 0. It joins the asymptote Qr = 0 for tc > 1 s.

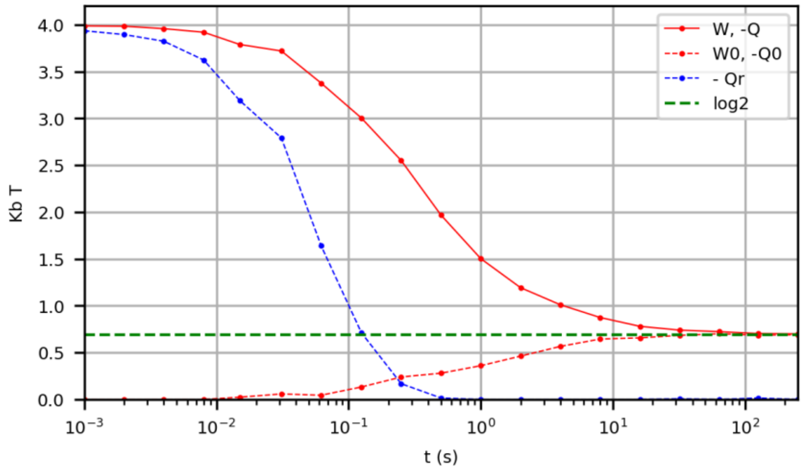

Figure 7. *Evolution of W and - Q according to the duration of the cycle*

The curve (W0, -Q0) corresponds to cases where the initial state is 0. When the cycle time is reduced, the exchange of energy and heat then decreases regularly and tends towards 0. The asymptote is reached around tc ~10ms. We can then create a bit without spending any energy, as we will see in more detail in section 5.

4. Realization of Maxwell's Demons

Tilt memory makes it possible to create a Maxwell demon capable of transforming thermal energy into active energy (mechanical, electrical or magnetic) much greater than k_bT per cycle.

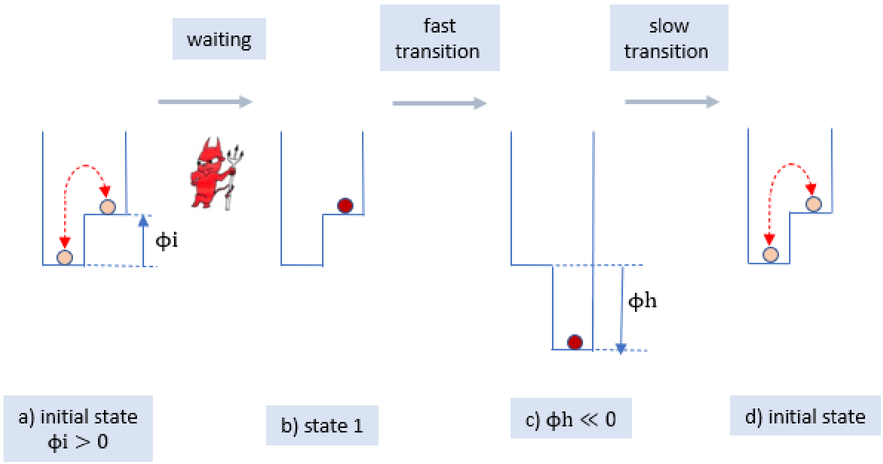

Figure 8. *A Maxwell's Demon*

A possible cycle for such a demon ([RiRi19]) is the following (figure 8):

a) In the initial state, the energy difference is fixed at the configurable value $\phi_i > 0$, so that the bit is generally at 0. The probability of state P_1 is then, according to equation (3): $P_{1i} = \frac{1}{1+e^{\phi_i}}$.
The "demon" watches for a random jump to state 1.

b) As soon as the demon detects a jump to state 1, the actuator suddenly switches ϕ from the positive value ϕ_i to the large negative value ϕ_h, which stabilizes the memory at state 1. The corresponding probability P_{1h} is: $P_{1h} = \frac{1}{1+e^{\phi_h}}$. For $\phi_h = -8$, we have $P_{1h} = 0{,}9997$. We round off to $P_{1h} = 1$.

During this adiabatic phase, there is no thermal exchange, so: $\Delta W_1 = \Delta U$.
As $\Delta U = \phi_h - \phi_i$, we get: $\Delta W = \phi_h - \phi_i$,

or according to (5): $\Delta W_1 = \log\frac{1-P_{1h}}{P_{1h}} - \log\frac{1-P_{1i}}{P_{1i}}$

c) The actuator slowly follows the reverse path. When it reaches $\phi = \phi_i$, according to equation (7) we have:
$\Delta W_2 = [\log(1-P_{1i})]_{P_{1h}}^{P_{1i}} = \log(1-P_{1i}) - \log(1-P_{1h})$

The total energy transferred from the actuator to the memory is finally:

$$\Delta W = \Delta W_1 + \Delta W_2 = \log P_{1i} - \log P_{1h}$$

$$\Delta W = \log P_{1i}$$

This value is negative which means that during the cycle the thermal energy $\Delta Q = -\log P_{1i}$ was transformed into active energy returned to the actuator. This energy is theoretically unlimited when P_{1i} tends towards 0 but the frequency of the event then tends to 0 exponentially.

It should be noted that this energy is only subtracted from the energy of the actuator, whether electrical or mechanical, and is not used outside the system. Koski et al. ([KKKA15]) made a refrigeration system by connecting two such memories, one of which pumped heat to the other. However it is probable that in all cases the power involved remains much lower than that which is dissipated in the actuator or in the whole of the experimental device.

5. Creation of a bit of information without energy

The above model is validated by all the experiments carried out in recent years. It shows that the transitions of these two-state memories are perfectly explained by the equations of classical physics, without any reference to quantum physics concepts.

However, it brings to light a strange aspect of the notion of entropy in this field of very low energies. For example (Figure 9) there are, in a tilt memory, several possible paths to pass from the initial random state $P_1 = 0.5$ to the final state $P_1 = 0$ (bit = 0, $\phi_e = \phi_h$), with a high ϕ_h value. We will consider two paths whose energy balances are very different.

A. We can increase slowly ϕ_e quasi-statically, that is to say for a period $t \gg t_0$ with $t_0 = \frac{1}{f_0} \sim 1\, s$. The actuator then expends energy $\Delta W = \log 2$ to create this bit. This energy is transformed into heat, transmitted to the thermostat, and decreases the entropy of the memory by $\Delta S = \log 2$. The result is independent of the initial state of the memory, whether it is 0 or 1.
This experiment was carried out in particular by [KMPA14] with a single electron.

B. We can increase ϕ_e at the moment when the bit is at 0, suddenly, that is to say over a period $t << t_0$. In this case, no elementary transition generally takes place during this short instant. The change of state is adiabatic; it does not require any energy. We have therefore obtained the same decrease in entropy as by the previous path, but in this case without energy expenditure.
This experiment was carried out in particular with a single electron by [KMPA14] and with a bistable molecule by [RiRi19].

All intermediate cases between A and B are possible by varying t.

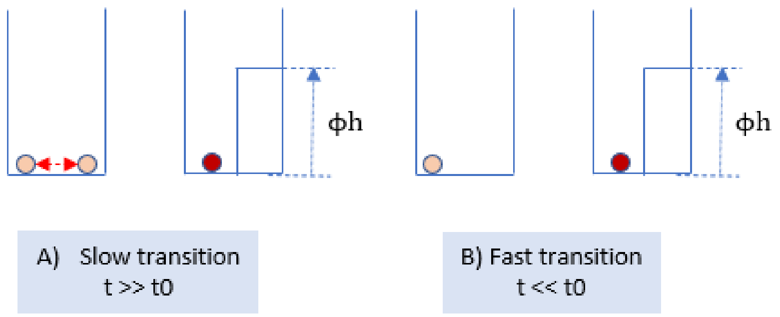

Figure 9. *Two paths to create a bit*

The second path (B) thus allowed us to pass from the random state of the memory to a stable bit with neither expenditure of energy nor variation of entropy of the thermostat. By changing the configuration of the system, we have reduced its entropy without expending energy. This fact could at first be surprising. However, it is validated by the experiments cited above ([KMPA14] , [RiRi19]).

We have produced some *negentropy* in the sense that Brillouin uses it ([Bril59]), for whom there is equivalence between a bit of information and a negentropy Kb log 2. In our example, the new configuration of the system has made it capable of transforming thermal energy into active energy.

6. Temporality of entropy

To try to understand how it is possible to change the entropy without energy exchange, consider the process of Figure 10:

a) In the initial state $\phi_h >> 0$ memory is stable at state 0. Its entropy is $S = -\log 2$. We suddenly lower ϕ_h to 0. This operation is adiabatic, energetically neutral.

b) During a certain time t << t_0, with t_0 the average switching time of the system at equilibrium, the system remains stable in state 0. Its entropy does not change.

c) After some time t >> t_0, the memory is randomized, switching between 0 and 1 at the average frequency $f_0 = 1/t_0$. Its entropy has therefore increased by $\log 2$ with respect to state (a), while its potential energy has not changed, and there has been no exchange of energy between the memory and the thermostat.

During the experiment, memory entropy increased by $\log 2$ with no energy transfer, which contradicts the classical definition of entropy, that $dS = dQ/T$.

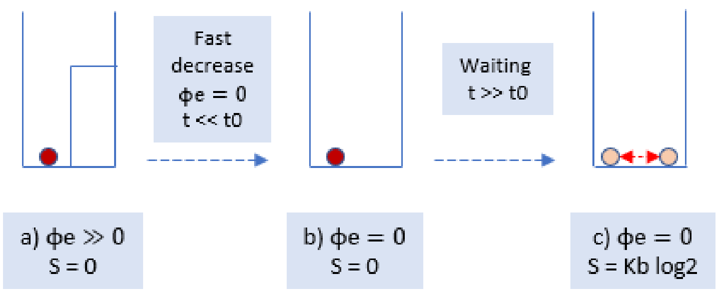

Figure 10. *Entropy paradox*

Landau and Lifshitz give us elements to understand this paradox ([LaLi69] p.26): “Attention should be drawn to the significance of time in the definition of entropy. The entropy is a quantity which describes the average properties of a body over some non-zero interval of time Δt. If Δt is given, to determine S we must imagine the body divided into parts so small that their relaxation times are small in comparison with Δt. Since these parts must also themselves be macroscopic, it is clear that when the intervals Δt are too short, the concept of entropy becomes meaningless; in particular, we cannot speak of its instantaneous value".

This caveat applies in the present case, since entropy is evaluated at different time scales in states (b) and (c).

The fact remains that this state of affairs is strange, and that it joins the questions of certain physicists on the notion of entropy. According to Penrose ([Penr07] p.670): "My own position concerning the physical status of entropy is that I do not see it as an 'absolute' notion in present-day physical theory, although it is certainly a very useful one. There is, however, the possibility that it might acquire a more fundamental status in the future. For this, quantum physics would certainly need to be taken into consideration …".

This strangeness could therefore come from a link between entropy and quantum physics, which Landau and Lifshitz would not contradict ([LaLi69] p.31) "It is more reasonable to suppose that the law of increase of entropy [...] arises from quantum effects."

7. Szilard's Machine

Szilard's machine has often served as a reference for the analysis of the links between information and entropy. Szilard came up with this thought experiment in an attempt to solve the paradox of Maxwell's demon ([Szil29]). After the foregoing discussion we can see it as an example of Maxwell's demon, and we can explain it within the framework of classical physics.

Consider the symmetric Szilard machine (figure 11) proposed in [Argo21] . A cylinder containing a single molecule of gas is equipped with a mobile valve that can divide the cylinder into two equal parts by sliding vertically, and that can move horizontally from one end of the cylinder to the other with negligible friction. Two weights are connected on either side of the valve by cables. At equilibrium they rest on a horizontal surface, and one of them rises as soon as the valve leaves the middle position.

We assume that the sealing of the valve is not complete when it is in the closed position, so that the molecule can cross it with an average frequency f_1 close to zero.

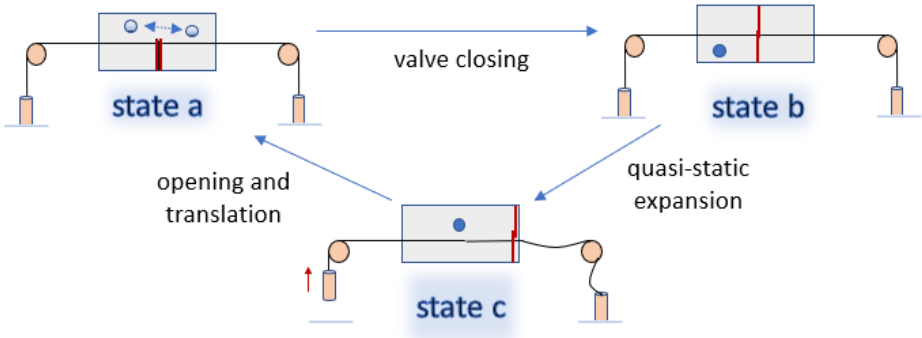

Figure 11. *Symmetrical Szilard machine*

In state (a) the valve is open and the molecule moves from one half of the cylinder to the other under the effect of its thermal agitation. The valve is gradually closed by sliding its mobile part vertically. We are in the situation of a bistable memory whose barrier is gradually being raised. During closure the molecule can pass from one side of the valve to the other, making elementary transitions which are energy neutral since the two potential wells are at the same level. The frequency of these transitions decreases gradually until reaching a very low value f_1, at state (b). The molecule is then trapped on one side or the other of the valve for a duration $t << 1/f_1$.

If we then release the horizontal movement of the valve, the pressure of the molecule pushes to one end of the cylinder by lifting one of the two weights, getting to state (c). Each impact of the molecule on the valve transmits some mechanical energy to it, which is transformed into potential energy transmitted to the counterweight located on the same side as the molecule. Each impact slightly lowers the temperature of the molecule, immediately restored (in this quasi-static process) by the thermostat.

Between state (b) and state (c) the energies involved W= - $\log 2$ and Q = $\log 2$ correspond to the increase in entropy ΔS = $\log 2$ of the molecule ([Argo21]). If we could make this machine, it would be a Maxwell’s demon capable of transforming thermal energy into mechanical energy without any form of measurement or observation and without expenditure of energy. Szilard's machine was the origin of the confusion between entropy and information, when von Neumann resorted to the subjectivity of an observer to explain its operation ([Neum32]). According to von Neumann, it is the fact that an observer knows the position of the molecule which reduces its entropy and which makes it possible to extract energy from it. It is finally the paradox of the temporality of entropy which resolves the question.

8. Conclusion

We have established the equations which describe the changes of state of bistable or tilt memories, which are caused by the tunnelling effect or by thermodynamic fluctuations, without resorting to information theory or quantum physics. For quasi-static processes, they are obtained in an very simple manner. For dynamic processes they only depend on the functions giving the state transition frequencies as a function of the energy difference between the two states and of the height of an optional barrier separating the two states.

The resulting model is confirmed by several experiments carried out over the past ten years, at a microscopic (electron, molecule) or mesoscopic (micro-bead or colloidal particle, nano-magnet) level. It shows that the Landauer energy $\Delta W = k_B T \log 2$ is dissipated during the inversion of a bistable memory, which is irreversible. In a tilt memory, on the contrary, the operations of inverting and randomizing a bit are quasi-reversible, in the sense that this energy is almost completely recoverable.

The model is explicit for quasi-static state changes, and it allows direct numerical simulation in other cases. It makes it possible to calculate the amount of energy dissipated according to the duration of the change of state, and the efficiency of a Maxwell's demon using the tilt memory to transform thermal energy into active energy.

It confirms experimental results which may seem strange, since they show that it is possible to change a tilt memory from its random state to a stable bit, without energy expenditure, via a very fast adiabatic transformation. We can therefore produce negentropy by modifying the state of the memory, without supplying energy, and then use this negentropy to transform thermal energy into active energy.

The model gives a full explanation of Szilard's machine. Finally, it illustrates the entropy temporality paradox, according to which entropy, at this energy level, depends on the time interval used to evaluate it. It confirms a remark by Landau and Lifshitz dating from the middle of the previous century, and shows that the classical definition of entropy $dS = dQ/T$ is not directly applicable at this energy level.

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Appendix Evolution curves of Q and W – numerical simulation

The algorithm below is used to simulate the out-of-equilibrium evolution of memory. It uses only equations (8).

It is here directly applied to the case described in section 3.3, namely the creation of a stable state 0 (ϕh >> 0) from an equilibrium state ϕe = 0 during a cycle time tc.

```
ti += dt                                    # time increment
fi  = fih*ti/tc                             # ϕ = ϕh*time/cycle time
if bit == 1:
    W += fih*dt/tc                          # W increment, only if bit = 1
if bit == 0:                                # transition frequency 0=>1
    Fi = f0/exp(fi/2)                       # equation (8)
else:
    Fi = f0/exp(-fi/2i)                     # transition frequency 1=>0
pt = Fi*dti                                 # transition probability
x = random()                                # x random between 0 et 1
if x <= pt:                                 # random draw
    if bit == 0:                            # elementary transition
        bit = 1
        dQ  = fi                            # thermal transfer
    else:
        bit = 0
        dQ  = -fi
    Q += dQ                                 # Q increment
```

Box 1. *Algorithm of elementary transitions for a time increment dt*

By taking f_0 = 1 Hz, for the duration t_c= 200 s we reach the asymptote of figure 5. For 1000 simulations corresponding to the creation of a bit in a tilt memory we obtain the following distribution of W (figure A1, number of occurrences on the ordinate), which shows that the correspondence between the quantity of information of a bit and the related energy is purely statistical.

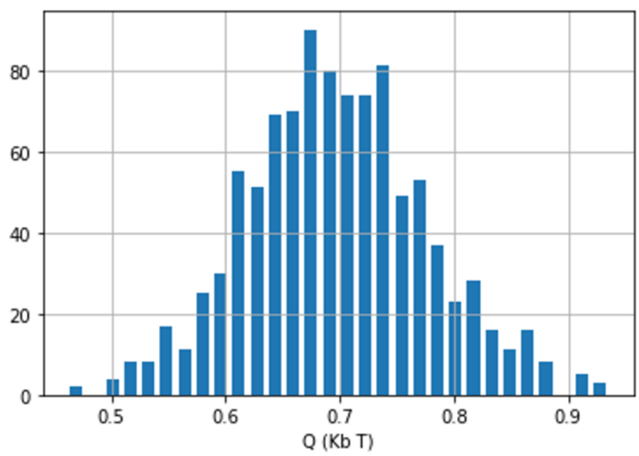

Figure A1. *An example of distribution of W*

